Nonstationary and Nonlinear Time Series Analysis using the Hilbert-Huang Transform

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Outline

- Introduction
- The Empirical Mode Decomposition (EMD) method, sifting
- Intrinsic Mode Function (IMF) components, the adaptive basis through EMD
- Confidence limit, degree of stationarity, and statistical significance of IMF
- A different view on nonlinearity
- Applications and examples
- Limitations of HHT and unfinished work
- Contact information

Intro: Motivations

- Physical processes are mostly nonstationary
- Physical processes are mostly nonlinear
- Data from observations are invariably too short
- Physical processes are mostly nonrepeatable

∪ Ensemble mean impossible, and temporal mean might not be meaningful for lack of ergodicity. Traditional methods are inadequate.

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Intro: Available Data Analysis Methods for Nonstationary (but Linear) Time Series

- Various probability distributions
- Spectral analysis and spectrogram
- Wavelet analysis
- Wigner-Ville distributions
- Empirical orthogonal functions (aka singular spectral analysis)
- Moving means
- Successive differentiations

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Intro: Available Data Analysis Methods for Nonlinear (but Stationary and Deterministic) Time Series

- Phase space method
  - Delay reconstruction and embedding
  - Poincaré surface of section
  - Self-similarity, attractor geometry & fractals
- Nonlinear prediction
- Lyapunov exponents for stability

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Intro: Consequences of these Methods

- With the explosion of data and computer, the field is ready for a data analysis methodology revolution.
- We not only need new methods but also a new paradigm for analyzing data from nonlinear and nonstationary processes.

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Intro: History of EMD

1998: The Empirical Mode Decomposition Method and the Hilbert Spectrum for Non-stationary Time Series Analysis, *Proc. Roy. Soc. London*, A454, 903-995.
The introduction of the basic method of EMD and Hilbert transform for determining the instantaneous frequency and energy.

1999: A New View of Nonlinear Water Waves – The Hilbert Spectrum, *Ann. Rev. Fluid Mech.* 31, 417-457.
Introduction of the intermittence in EMD decomposition.

2003: A confidence Limit for the Empirical mode decomposition and the Hilbert spectral analysis, *Proc. of Roy. Soc. London*, A459, 2317-2345.
Establishment of a confidence limit without the ergodic assumption.

2004: A Study of the Characteristics of White Noise Using the Empirical Mode Decomposition Method, *Proc. Roy. Soc. London*, (in press)
Defined statistical significance and predictability for IMF from EMD.

2004: On the Instantaneous Frequency, *Proc. Roy. Soc. London*, (Under review)
Removal of the limitations posted by Bedrosian and Nuttall theorems for Instantaneous Frequency computations.

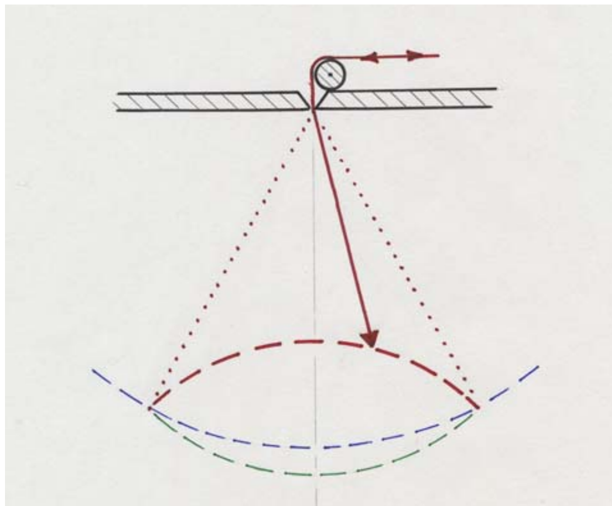
Intro: Characteristics of Data from Nonlinear Processes

$$\frac{d^2 x}{dt^2} + x + \varepsilon x^3 = \gamma \cos \omega t$$

$$\Rightarrow \frac{d^2 x}{dt^2} + x \left(1 + \varepsilon x^2 \right) = \gamma \cos \omega t$$

$\Rightarrow$ *Spring with position dependent constant,*
intra-wave frequency modulation;
therefore, we need instantaneous frequency.

Intro: Duffing Pendulum



Intro: Definition of Hilbert Transform

For any $x(t) \in L^p$,

$$y(t) = \frac{1}{\pi} \wp \int_{\tau} \frac{x(\tau)}{t-\tau} \, d\tau \ ,$$

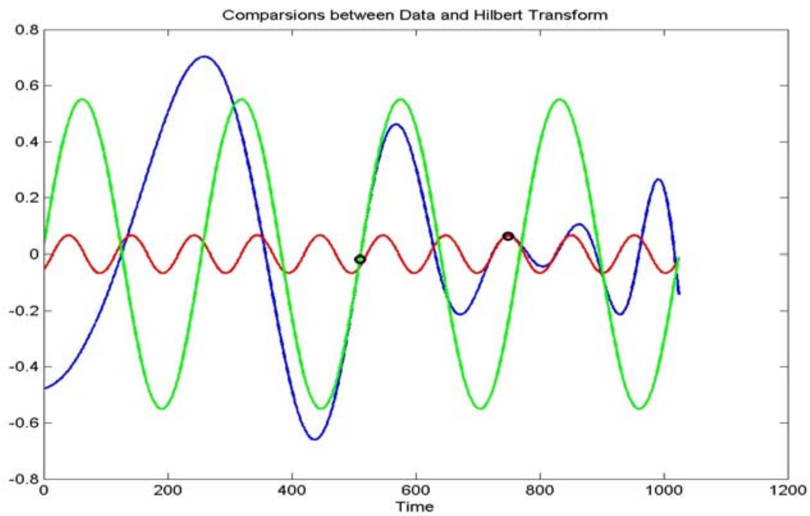
then, $x(t)$ and $y(t)$ are complex conjugate :

$$z(t) = x(t) + i\, y(t) = a(t)\, e^{i\theta(t)} \ ,$$

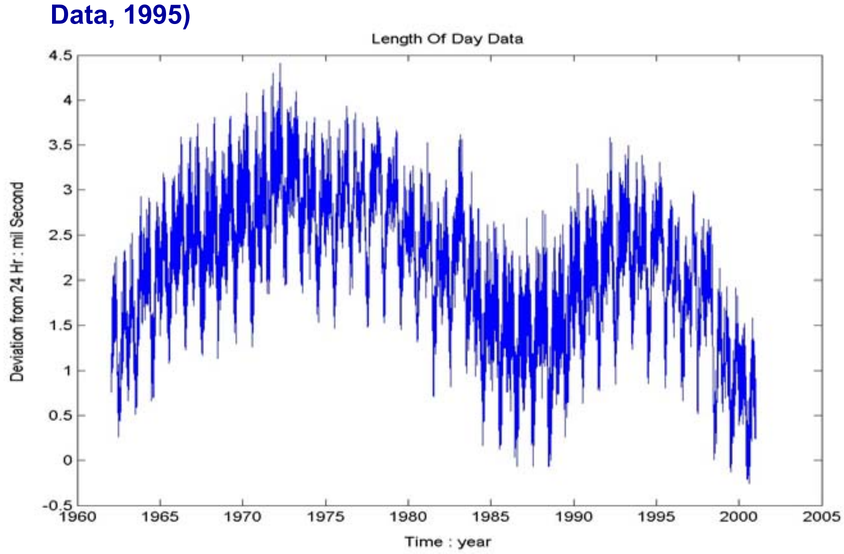
where

$$a(t) = \left(x^2 + y^2\right)^{1/2} \ \text{and} \ \ \theta(t) = \tan^{-1} \frac{y(t)}{x(t)} \ .$$

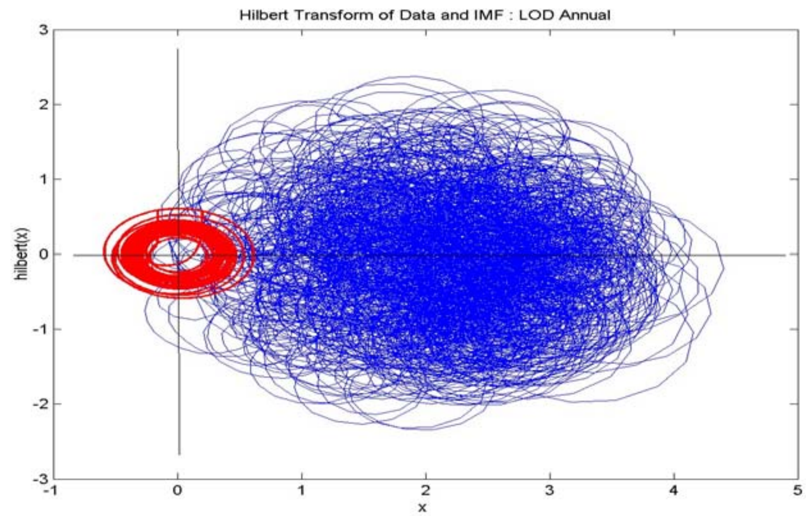
Intro: Hilbert Transform Fit



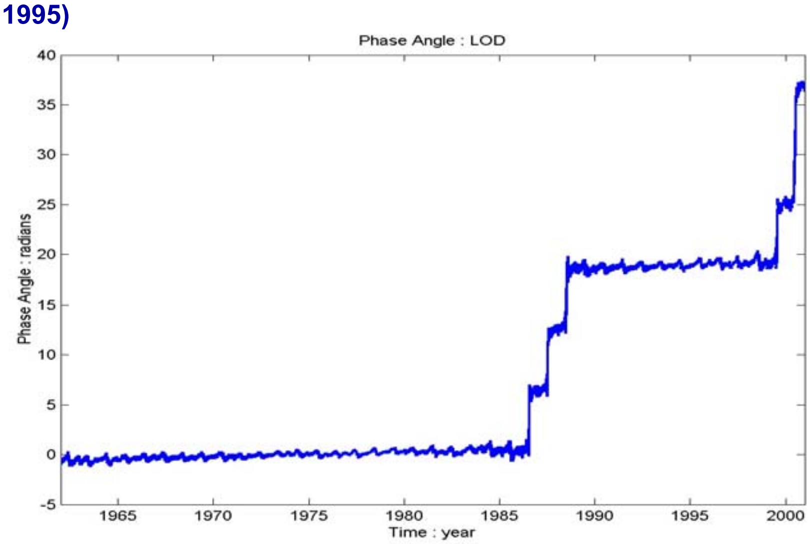
Intro: Traditional View **a la Hahn (Length of Day Data, 1995)**



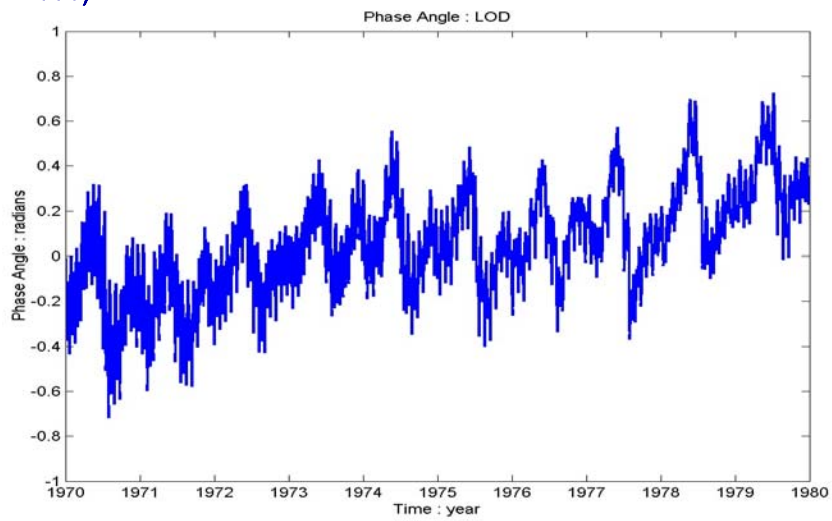
Intro: Traditional View **a la Hahn (Hilbert, 1995)**



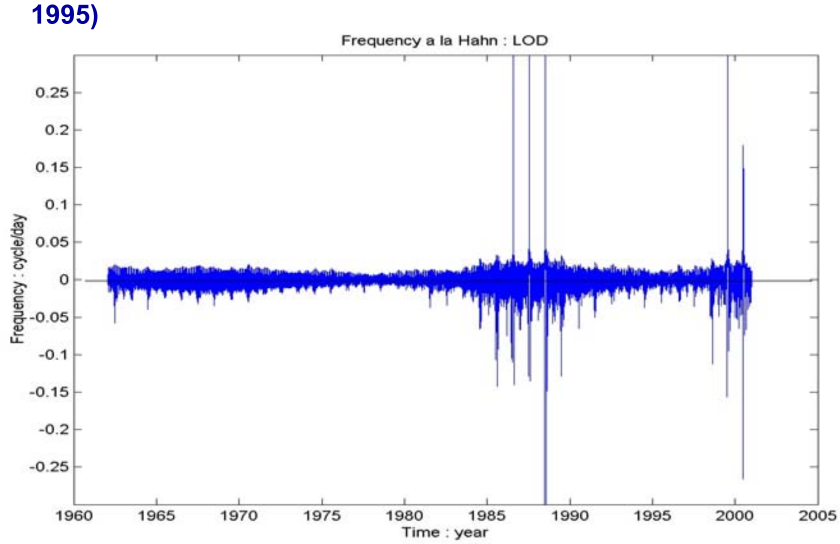
Intro: Traditional View **a la Hahn (Phase Angle, 1995)**



Intro: Traditional View a la Hahn (Phase Angle, 1995)



Intro: Traditional View a la Hahn (Frequency, 1995)



Why doesn't the traditional approach work?

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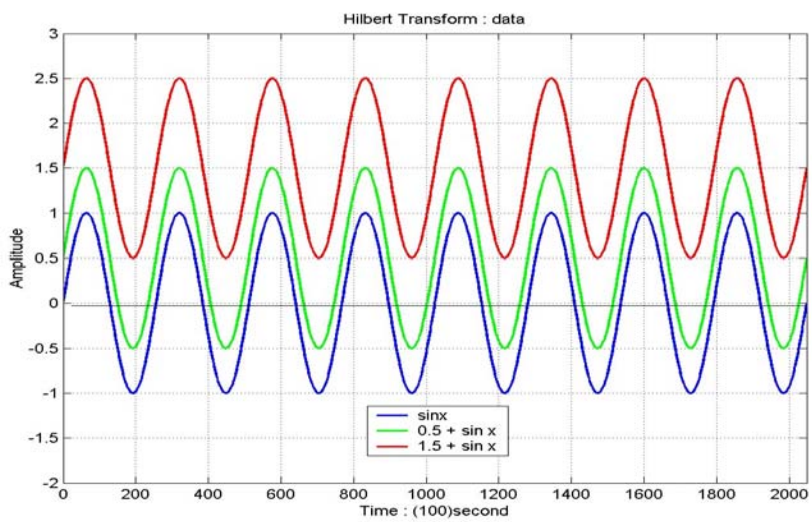
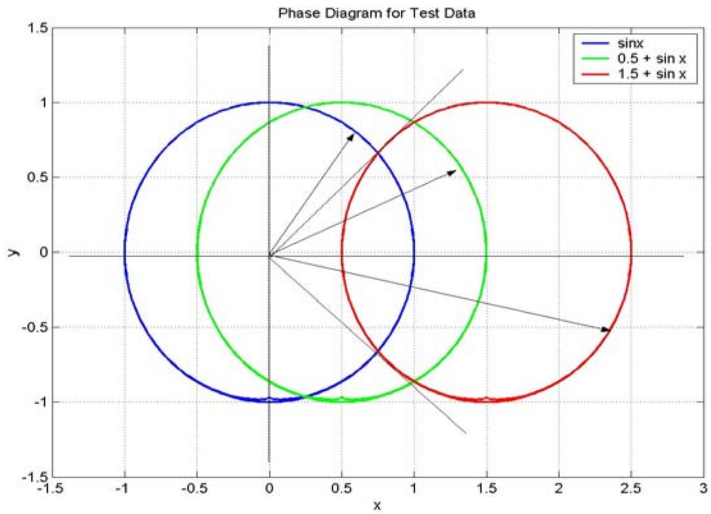
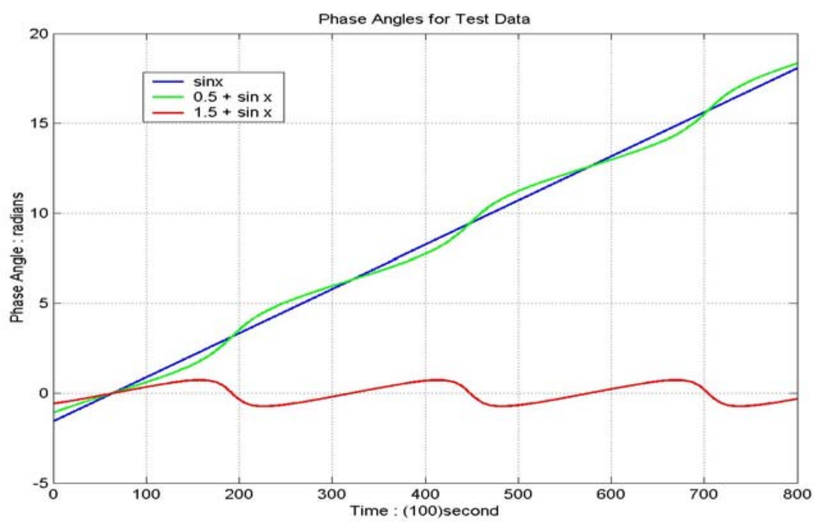
Intro: Why the traditional view doesn't work… Hilbert Transform a cos θ + b (Data)



Intro: Why the traditional view doesn't work…
Hilbert Transform a cos θ + b (Phase Diagram)

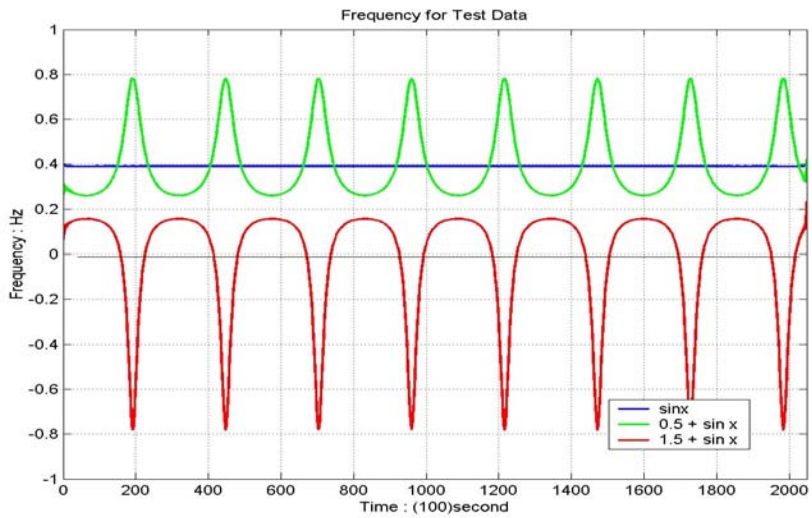


Intro: Why the traditional view doesn't work…
Hilbert Transform a cos θ + b (Phase Angle Details)



Intro: Why the traditional view doesn't work…

Hilbert Transform a cos θ + b (Frequency)



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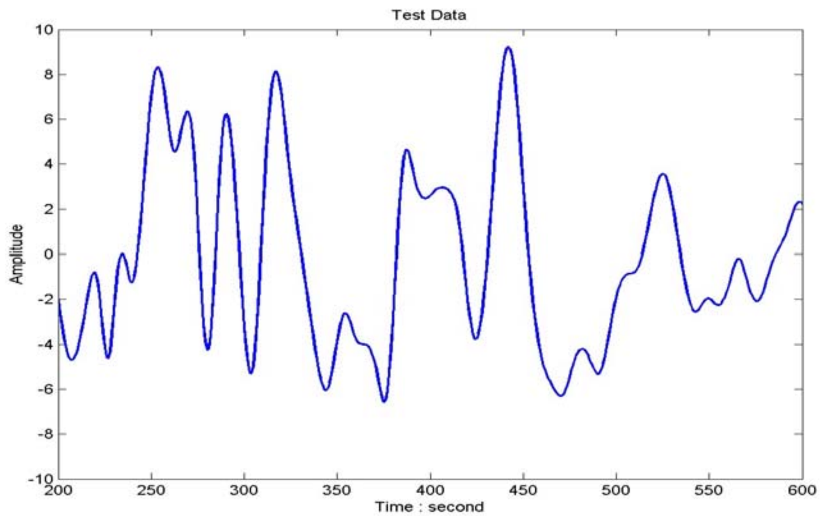
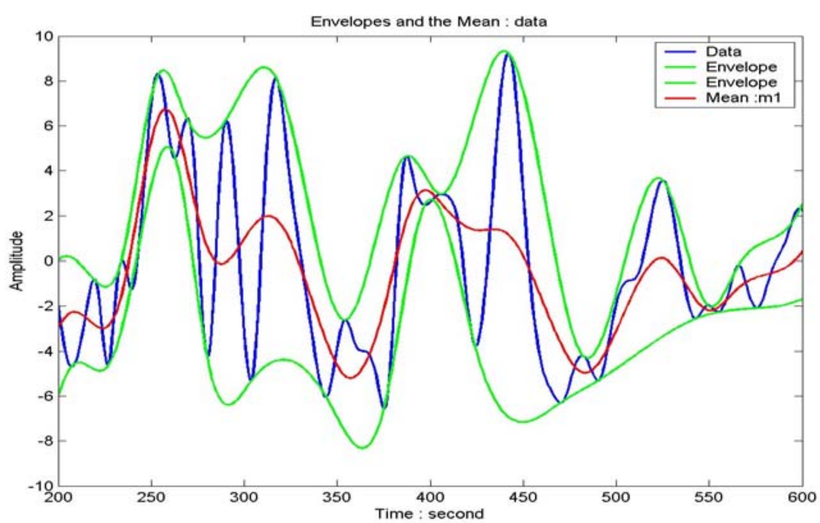
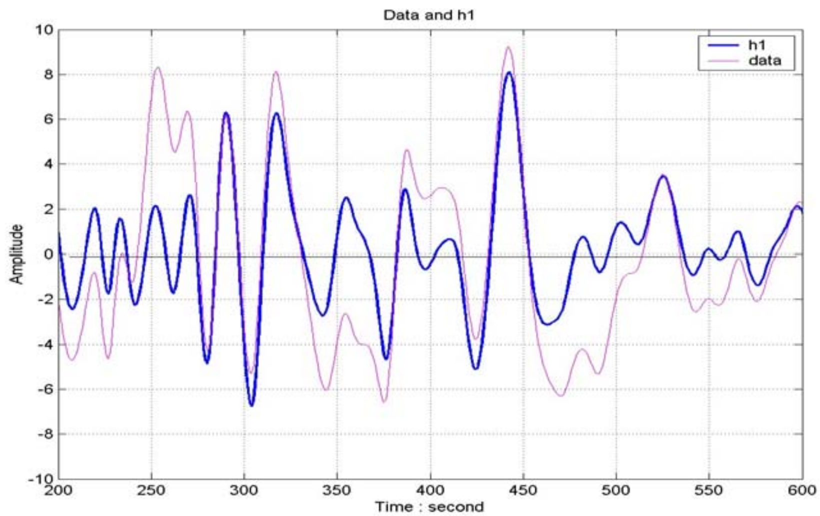
EMD & Sifting: Test Data



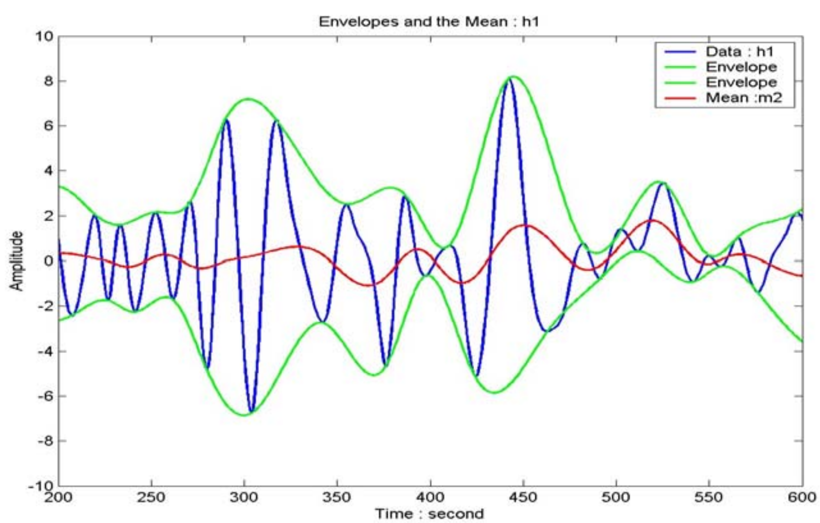
EMD & Sifting: Test Data and Mean M1



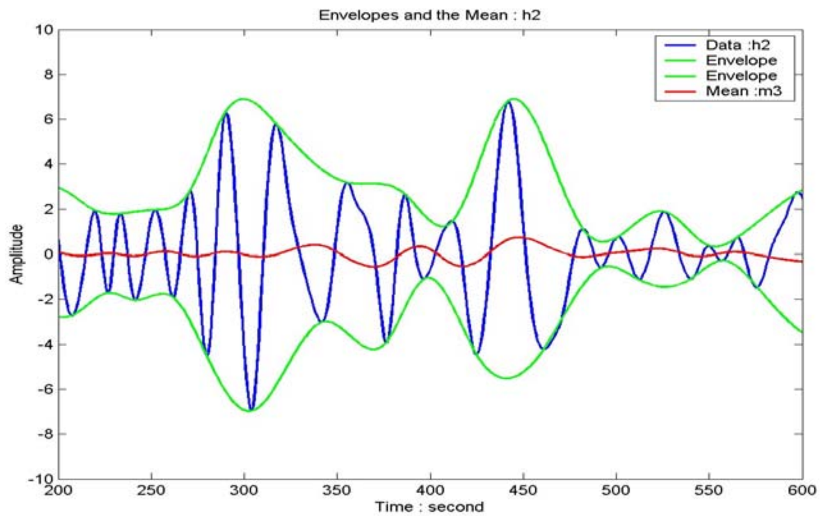
EMD & Sifting: Test Data and H1



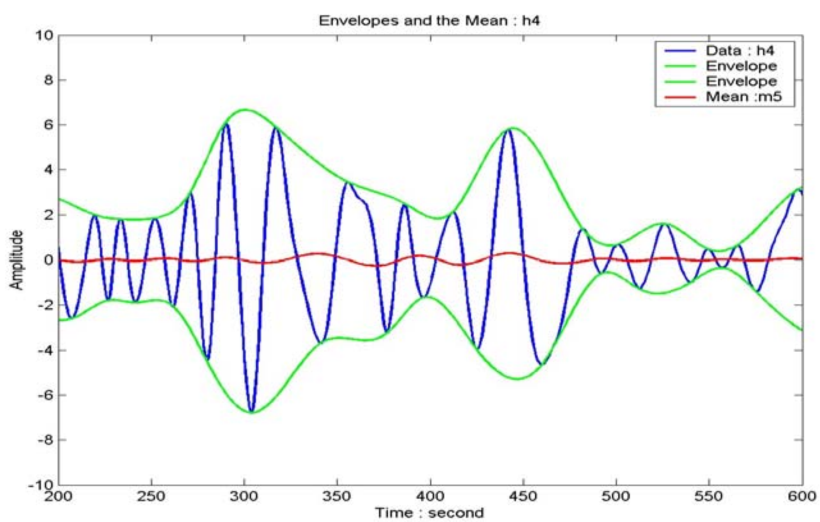
EMD & Sifting: Test Data, H1, Mean M2



EMD & Sifting: Test Data, H2, Mean M3



EMD & Sifting: Test Data, H4, Mean M5



EMD & Sifting: Getting one IMF Component

Sifting : to get one IMF component

$$x(t) - m_1 = h_1 ,$$
$$h_1 - m_2 = h_2 ,$$
$$\cdots\cdots$$
$$\cdots\cdots$$
$$h_{k-1} - m_k = h_k .$$

$$\Rightarrow \quad h_k = c_1 \quad .$$

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EMD & Sifting: Two Stoppage Criteria (S and SD)

A. The S number : S is defined as the consecutive number of siftings in which the number of zero-crossing and extrema are the same for these S siftings.

B. SD is small than a pre-set value, where

$$SD = \sum_{t=0}^{T} \frac{\left| h_{k-1}(t) - h_k(t) \right|^2}{h_{k-1}^{\,2}(t)} .$$

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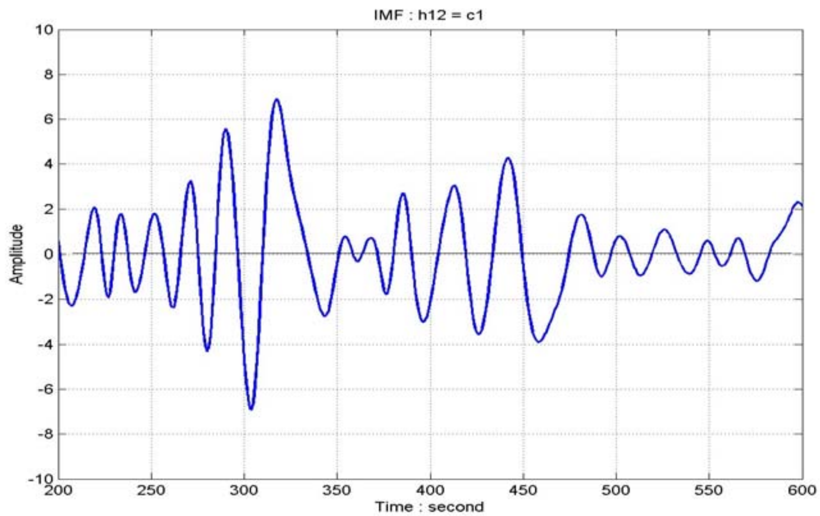
EMD & Sifting: IMF C1



EMD & Sifting: Definition of the Intrinsic Mode Function

Any function having the same numbers of zero – crossings and extrema, and also having symmetric envelopes defined by local maxima and minima respectively is defined as an Intrinsic Mode Function (IMF).

All IMF enjoys good Hilbert Transform :

$$\Rightarrow\Rightarrow \quad c(t) = a(t)e^{i\theta(t)}$$

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EMD & Sifting: Getting all IMF Components

Sifting : to get all the IMF components

$$x(t) - c_1 = r_1 \ ,$$

$$r_1 - c_2 = r_2 \ ,$$

$$\cdots$$

$$r_{n-1} - c_n = r_n \ .$$

$$\Rightarrow \ x(t) - \sum_{j=1}^{n} c_j = r_n \ .$$

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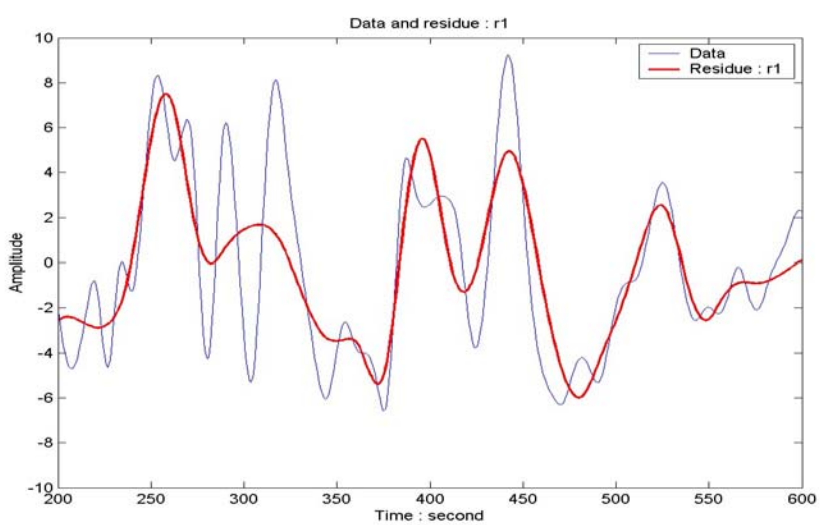
EMD & Sifting: Test Data and Residue R1



EMD & Sifting: Definition of Instantaneous Frequency

The Fourier Transform of the Instrinsic Mode Funnction, c(t), gives

$$W(\omega) = \int_t a(t)\, e^{i(\theta - \omega t)}\, dt$$

By Stationary phase approximation we have

$$\frac{d\theta(t)}{dt} = \omega \,,$$

This is defined as the Instantaneous Frequency.

EMD & Sifting: Comparison between FFT and HHT

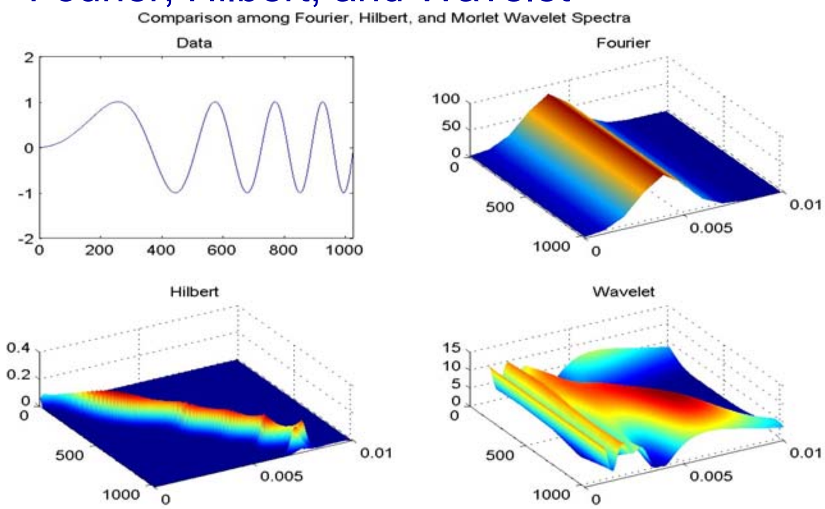
1. FFT :

$$x(t) = \Re \sum_j a_j e^{i\omega_j t} \,.$$

2. HHT :

$$x(t) = \Re \sum_j a_j(t)\, e^{i\int_t \omega_j(\tau)\, d\tau} \,.$$

EMD & Sifting: Comparisons Between Fourier, Hilbert, and Wavelet



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IMF Components: Adaptive Basis Generated by EMD

* Orthogonality †
* Completeness
* Uniqueness
* Convergence

These comprise the traditional check list.

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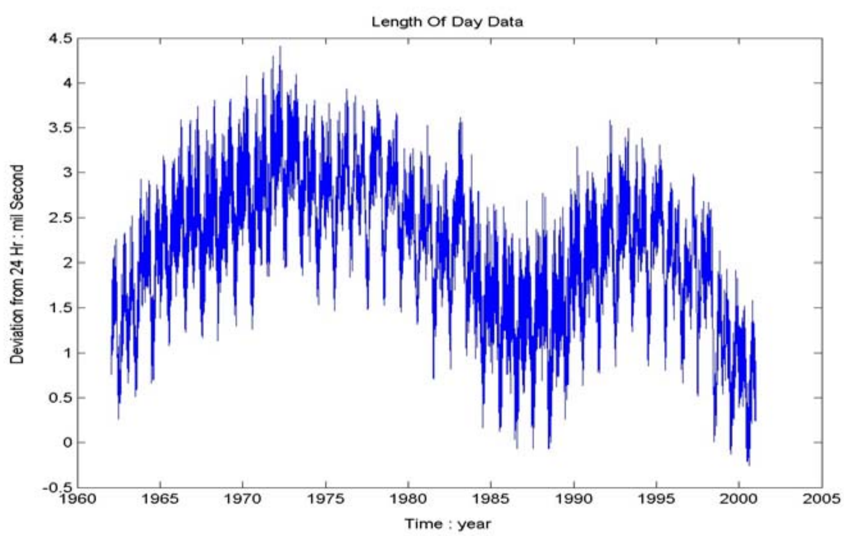
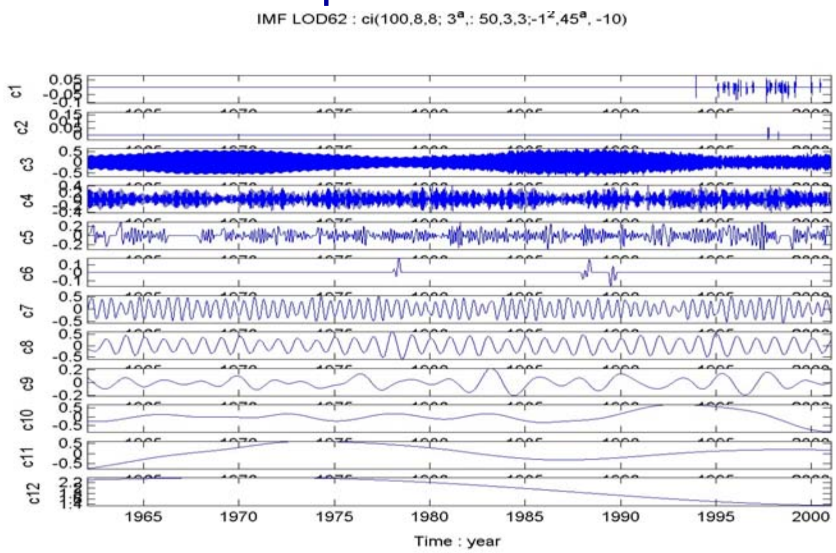
IMF Components: Length Of Day Data



IMF Components: LOD IMFs



IMF Components: Orthogonality Check

Pair-wise %

- 0.0003
- 0.0001
- 0.0215
- 0.0117
- 0.0022
- 0.0031
- 0.0026
- 0.0083
- 0.0042
- 0.0369
- 0.0400

Overall %

- 0.0452

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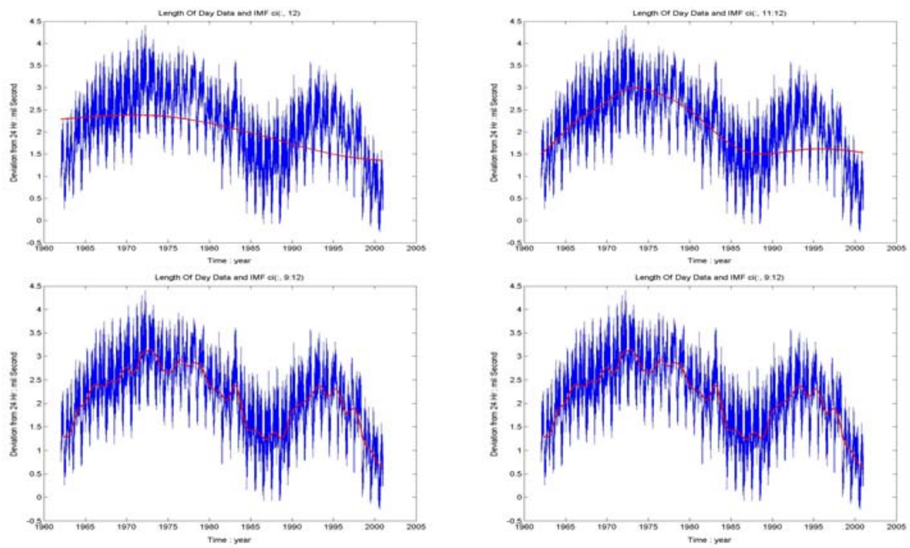
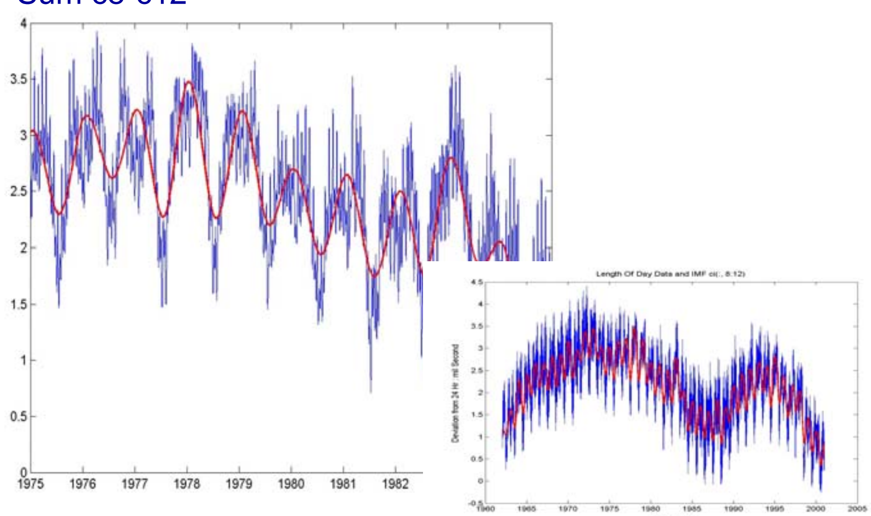
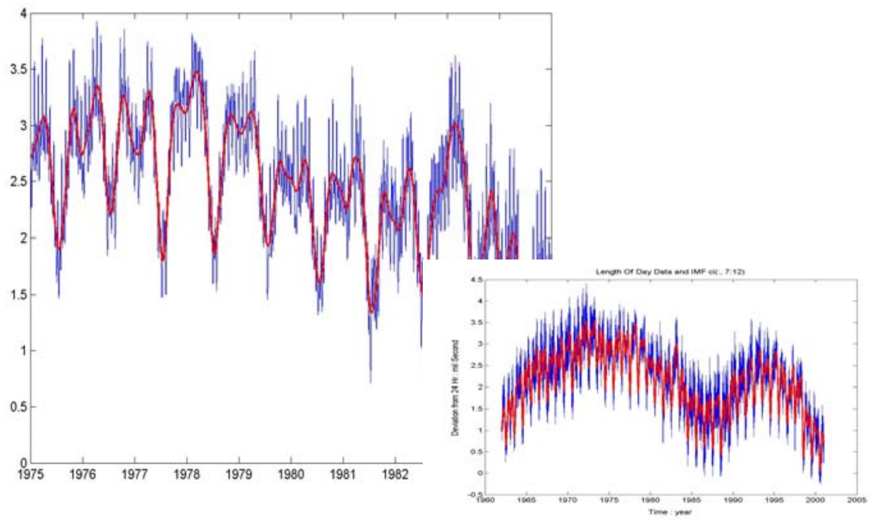
IMF Components: Data & Various Partial Sums



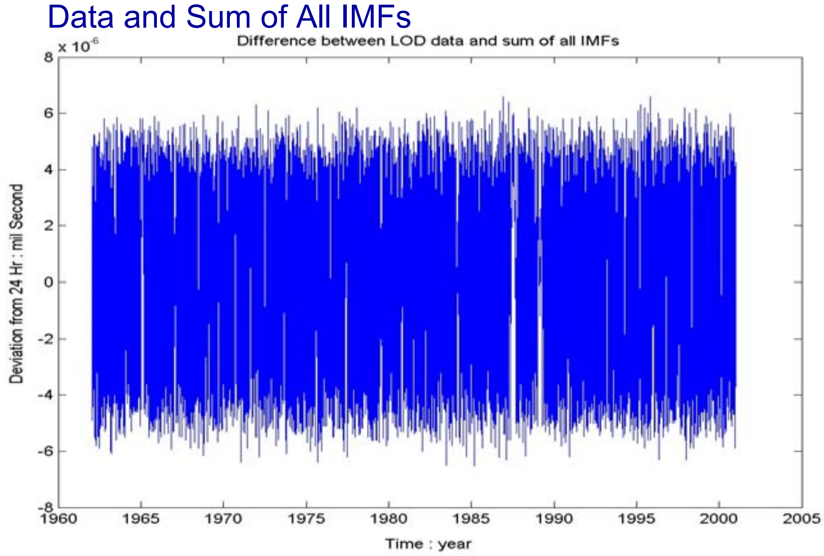
IMF Components: Detailed Length of Day Data and Sum c8-c12



IMF Components: Detail LOD Data and Sum IMF c7-c12



IMF Components: Difference Between LOD Data and Sum of All IMFs



IMF Components: EMD Generated Adaptive Basis

- Completeness
  - Given by definition
- Convergence
  - Simple reduced cases can be proven
- Orthogonality
  - Reynolds type decomposition: mean $\perp$ fluctuation; not necessary for nonlinear cases
- Uniqueness
  - With respect to adjustable parameters

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Confidence Limit: Confidence Limit for Fourier Spectrum

- The confidence limit for Fourier spectral analysis is based on ergodic assumption.
- It is derived by dividing the data into M sections and substituting the temporal (or spatial) average as the ensemble average.
- This approach is valid for linear and stationary processes, and the sub-sections have to be statistically independent.

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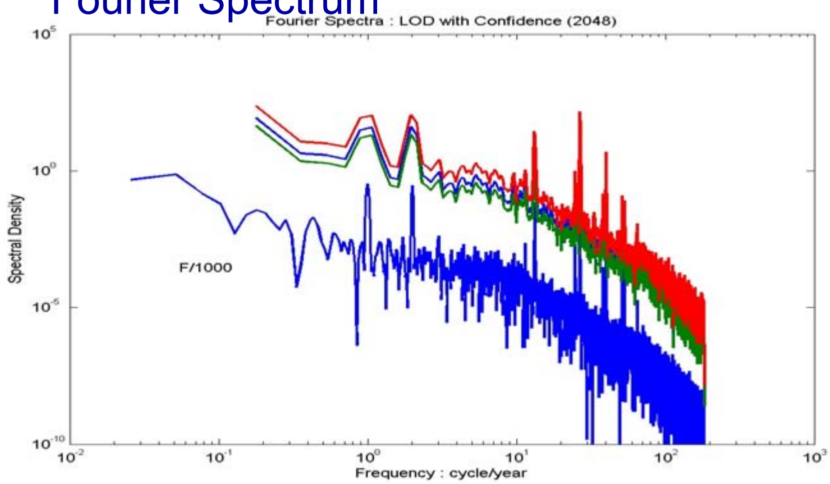
Confidence Limit: Confidence Limit for Fourier Spectrum



Confidence Limit from 7 sections, each 2048 points.

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Confidence Limit: Confidence Limit for Hilbert Spectrum

- Any data can be decomposed into infinitely many different component sets.
- EMD is a method to generate infinitely many different IMF representations based on different sifting parameters.
- Some of the IMFs are better than others based on various properties (e.g., Orthogonal Index).
- A confidence limit for Hilbert spectral analysis can be based on an ensemble of "valid" IMFs resulting from different sifting parameters S covering the parameter space fairly.
- It is valid for nonlinear and nonstationary processes.

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Confidence Limit: Critical Parameters for EMD

- N: the maximum number of siftings allowed to extract an IMF.
- S: the stoppage criterion, or criterion for accepting a sifting component as an IMF.
- Therefore, the nomenclature for the IMFs is as follows:

  CE(N, S) : for extrema sifting
  CC(N, S) : for curvature sifting

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Confidence Limit: Effects of EMD (Sifting)

- To separate data into components of similar scale
- To eliminate ridding waves
- To make the results symmetric with respect to the x-axis and to make the amplitude more even

  - Note: The first two are necessary for a valid IMF, the last effect actually caused the IMF to lose its intrinsic properties.

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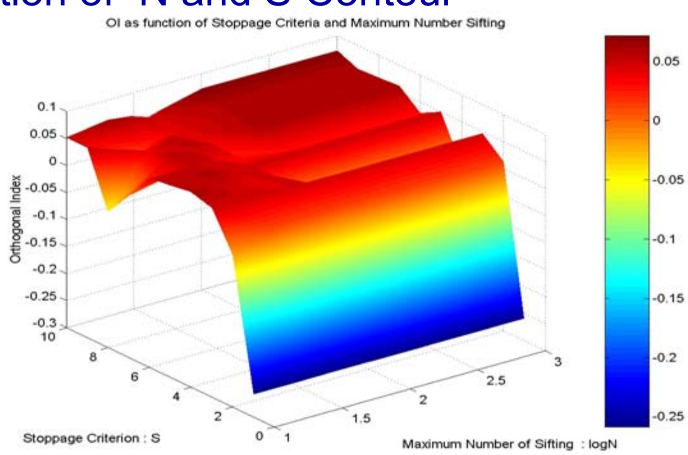
Confidence Limit: Orthogonal Index as Function of N and S Contour



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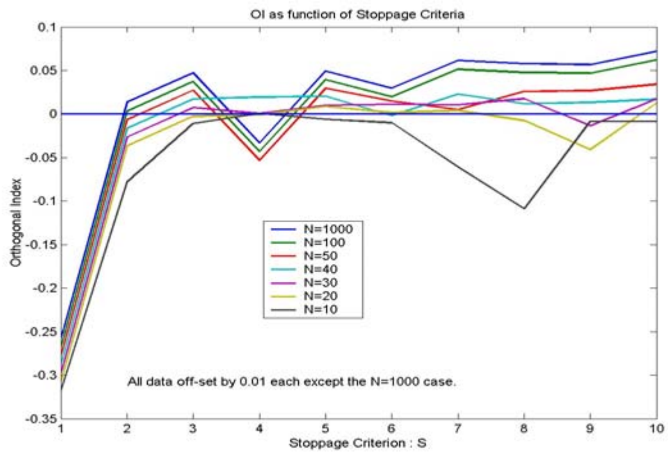
Confidence Limit: Orthogonality Index as Function of N and S



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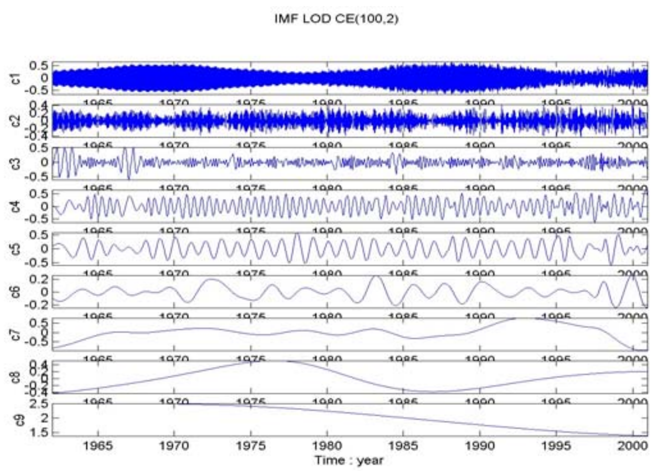
Confidence Limit: IMF CE(100, 2)



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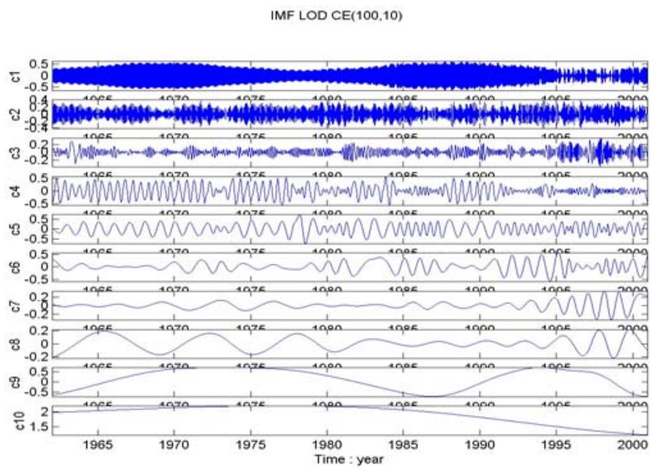
Confidence Limit: IMF CE(100, 10)



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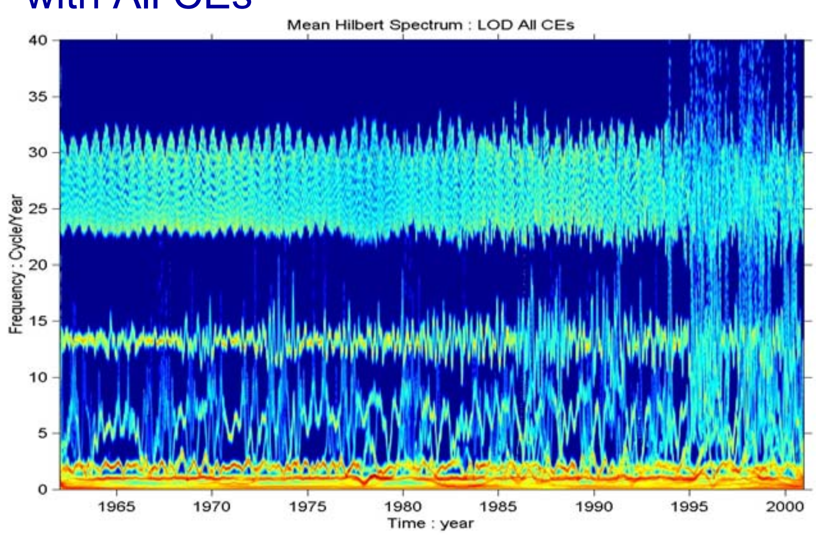
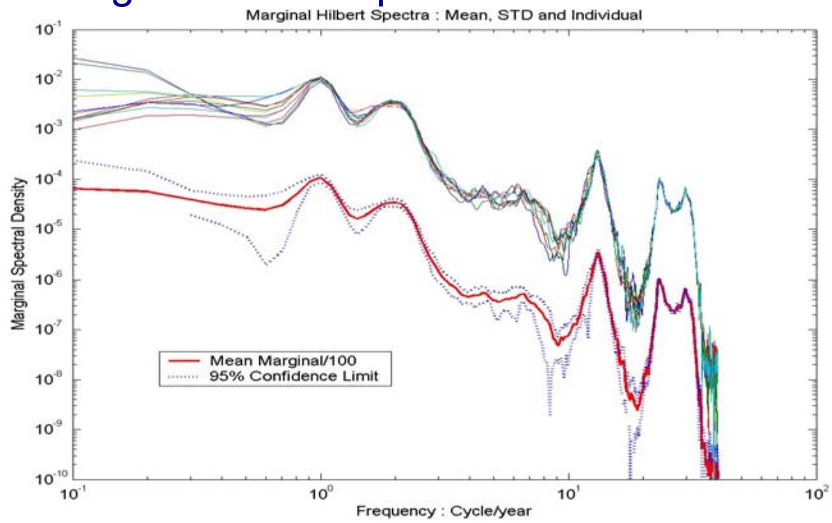
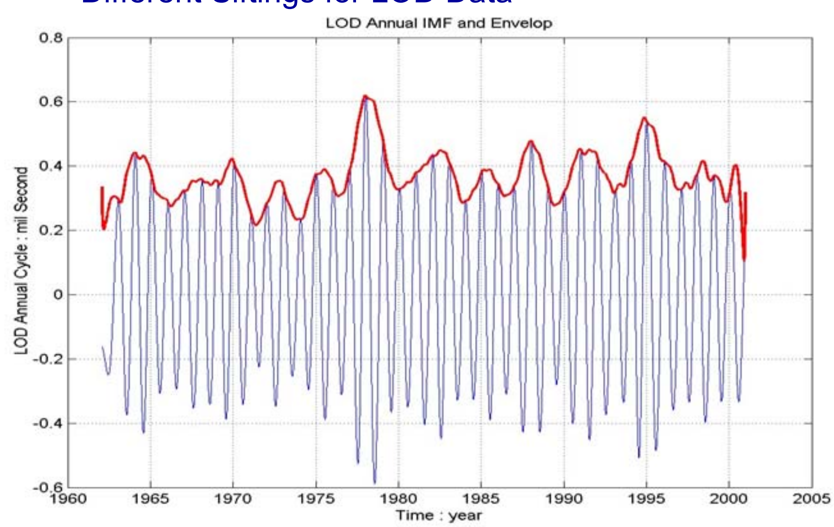
Confidence Limit: Mean Hilbert Spectrum with All CEs



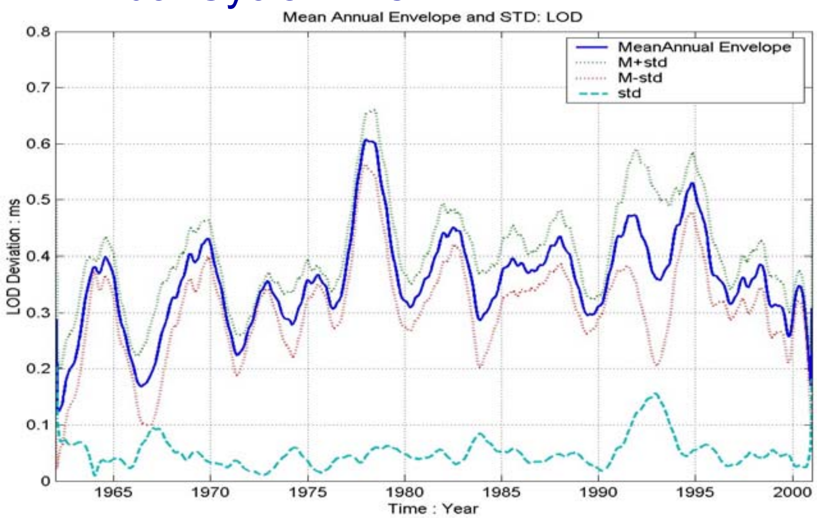
Confidence Limit: Mean and STD of Marginal Hilbert Spectra



Confidence Limit: Mean Envelope from 11 Different Siftings for LOD Data



Confidence Limit: Mean Envelopes for Annual Cycle IMFs



Degree of Staionarity: Defining the Degree of Stationarity

- Traditionally, stationarity is taken for granted; it is given; it is an article of faith.
- All the definitions of stationarity are too restrictive.
- All definitions of stationarity are qualitative.
- A good definition must be quantitative to give a Degree of Stationarity.

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Degree of Stationarity: Definition of Strict Stationarity

For a random variable $x(t)$, if

$$\langle |x(t)|^2 \rangle \prec \infty, \quad \langle x(t) \rangle = m, \quad \text{and that}$$

$$\left[x(t_1), x(t_2), \ldots x(t_n) \right] \text{ and } \left[x(t_1+\tau), x(t_2+\tau), \ldots x(t_n+\tau) \right]$$

have the same joint distribution for all τ.

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Degree of Stationarity: Definition of Wide Sense Stationarity

For any random variable $x(t)$, if

$$\langle |x(t)|^2 \rangle \prec \infty, \quad \langle x(t) \rangle = m, \quad \text{and that}$$

$$\left[x(t_1), x(t_2) \right] \text{ and } \left[x(t_1+\tau), x(t_2+\tau) \right]$$

have the same joint distribution for all τ.

$$\text{Therefore, } \langle x(t_1) \cdot x(t_2) \rangle = C(t_1 - t_2).$$

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Degree of Stationarity: Definition of Statistical Stationarity

- Applies if the stationarity definitions are satisfied with certain degree of averaging.
- All averaging involves a time scale. The definition of this time scale is problematic.

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Degree of Stationarity: For a Time-Frequency Distribution

For a time – frequency distribution, $H(\omega,t)$,

$$n(\omega) \equiv \frac{1}{T}\int_{t} H(\omega,t)\,dt \;;$$

$$DS(\omega) \equiv \frac{1}{T}\int_{0}^{T}\left[1-\frac{H(\omega,t)}{n(\omega)}\right]^{2} dt \,.$$

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Degree of Stationarity: Degree of Statistical Stationarity for a Time-Frequency Distribution

$$\text{For a time-frequency distribution, } H(\omega,t),$$

$$n(\omega) = \frac{1}{T}\int_t H(\omega,t)\,dt \;;$$

$$DS(\omega,\Delta t) = \frac{1}{T}\int_0^T \left[1-\frac{\langle H(\omega,t)\rangle_{\Delta t}}{n(\omega)}\right]^2 dt \;.$$

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Statistical Significance: Methodology

- Method is based on observations from Monte Carlo numerical experiments on 1 million white noise data points.
- All IMFs are generated by 10 siftings.
- Fourier spectra are based on 200 realizations of 4,000 data point sections.
- Probability densities are based on 50,000 data point data sections.

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Statistical Significance: IMF Period Statistics

IMF	1	2	3	4	5	6	7	8	9
Number of peaks	347042	168176	83456	41632	20877	10471	5290	2658	1348
Mean period	2.881	5.946	11.98	24.02	47.90	95.50	189.0	376.2	741.8
Periods in a year	0.240	0.496	0.998	2.000	3.992	7.958	15.75	31.35	61.75

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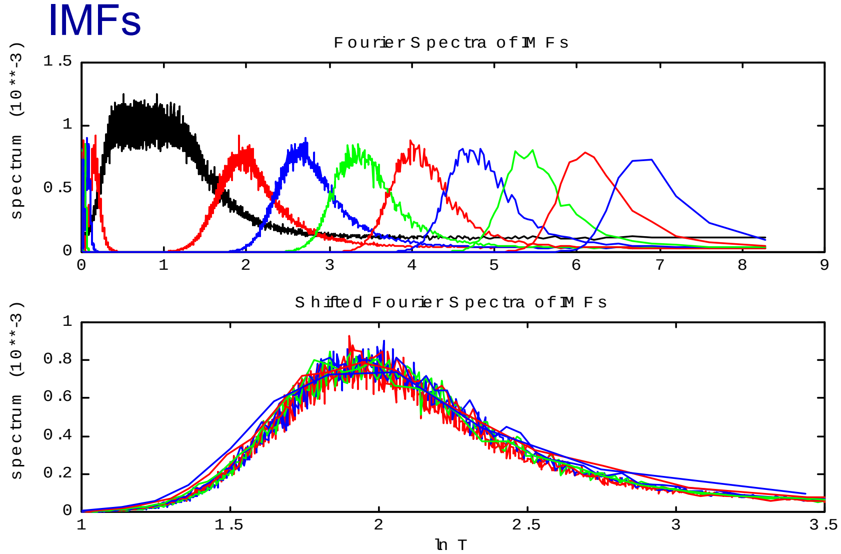
Statistical Significance: Fourier Spectra of IMFs



Statistical Significance: Empirical Observations I

Normalized spectral area is constant

$$\int S_{\ln T,n}\, d\ln T = const$$

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Statistical Significance: Empirical Observations II

Computation of mean period

$$NE_n = \int S_{\omega,n}\, d\omega = \int S_{T,n} \frac{dT}{T^2} = \int S_{\ln T,n} \frac{d\ln T}{T} = \frac{\int S_{\ln T,n}\, d\ln T}{\overline{T}_n}$$

$$\overline{T}_n = \frac{\int S_{\ln T,n}\, d\ln T}{\int S_{\ln T,n} \dfrac{d\ln T}{T}}$$

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Statistical Significance: Empirical Observations III

The product of the mean energy and period is constant

$$E_n \overline{T_n} = const$$

$$\ln E_n + \ln \overline{T_n} = const$$

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Statistical Significance: Monte Carlo Result (IMF Energy vs. Period)

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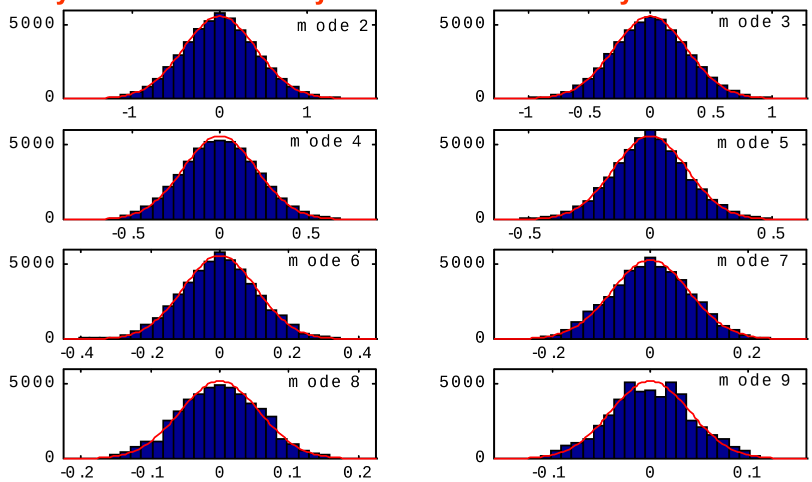
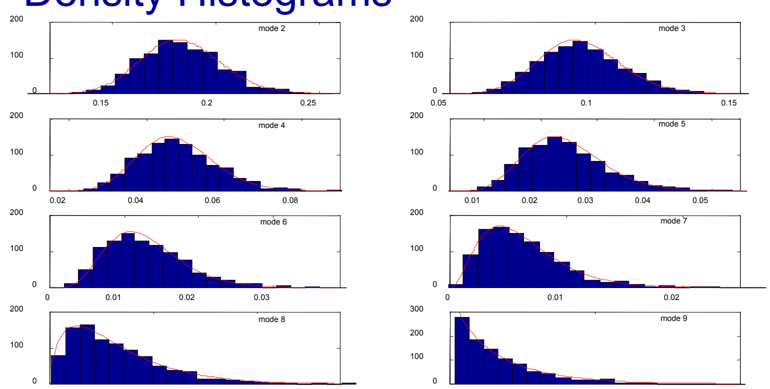
Statistical Significance: Empirical Observation, IMF Histograms

By Central Limit theory IMF should be normally distributed.



Statistical Significance: IMF Energy Density Histograms



By Central Limit Theory, the IMFs should be normally distributed; therefore, the energy density should be Chi-squared distributed.

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Statistical Significance: Chi-Squared Energy Density Distributions

By Central Limit Theory, the IMFs should be normally distributed; therefore, the energy density should be Chi-squared distributed.

$$\rho(E_n) = N \cdot (NE_n)^{N\overline{E}_n/2-1} e^{-NE_n/2}$$

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Statistical Significance: Formula for Confidence Limit for IMF Distributions

Introduce new variable y:

$$y = \ln E \qquad \text{Then,} \qquad E = e^y$$

$$\rho(y) = C \cdot \exp\left\{-\frac{N\overline{E}}{2}\left[1 - \overline{y} + \frac{(y-\overline{y})^2}{2!} + \frac{(y-\overline{y})^3}{3!} + \cdots\right]\right\}$$

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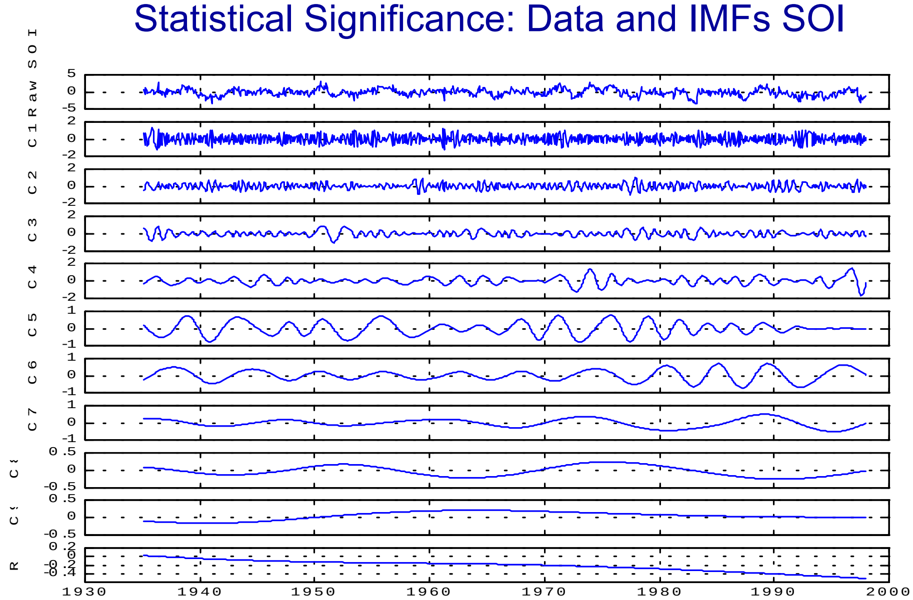
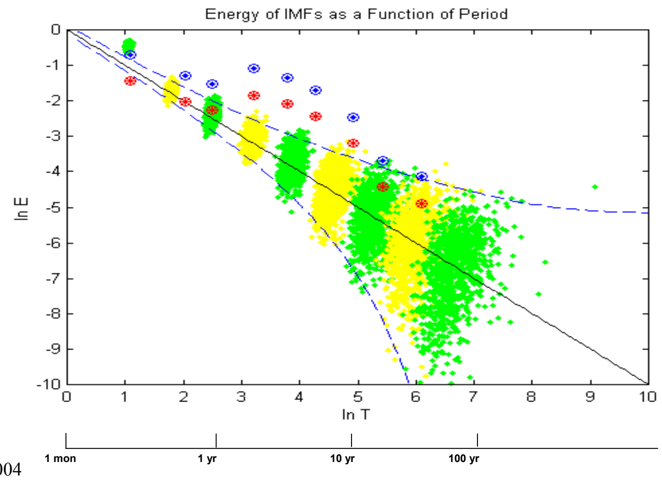
Statistical Significance: Confidence Limit for IMF Distributions

Statistical Significance: Data and IMFs SOI



Statistical Significance: Statistical Significance for SOI IMFs

IMFs 4, 5, 6 and 7 are 99% statistical significance signals.



Statistical Significance: Summary

- Not all IMFs have the same statistical significance.
- Based on the white noise study, we have established a method to determine the statistical significant components.
- References:
  - **Wu, Zhaohua and N. E. Huang, 2003: A Study of the Characteristics of White Noise Using the Empirical Mode Decomposition Method, Proceedings of the Royal Society of London (in press).**
  - **Flandrin, P., G. Rilling, and P. Gonçalvès, 2003: Empirical Mode Decomposition as a Filterbank, IEEE Signal Processing, (in press).**

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- Contact information

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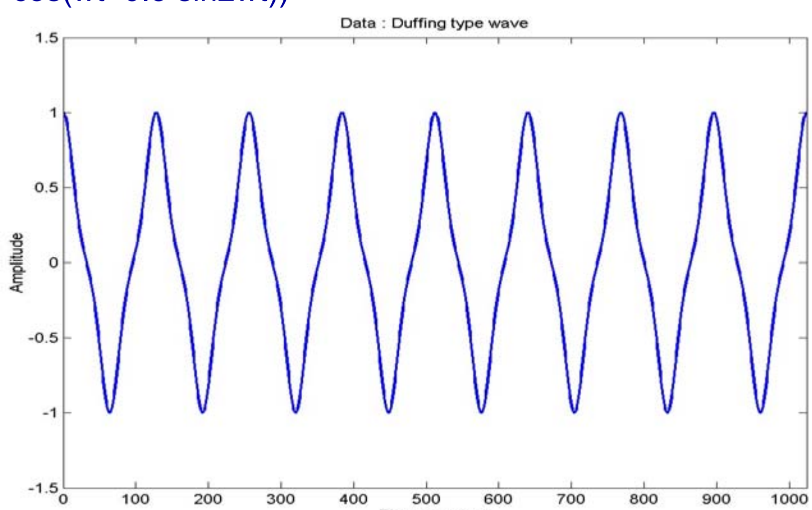
Nonlinearity: Duffing Type Wave (Data: x = cos(wt+0.3 sin2wt))



Nonlinearity: Duffing Type Wave
(Perturbation Expansion)

For $\varepsilon \ll 1$*, we can have*

$$
\begin{aligned}
x(t) &= \cos\left(\omega t + \varepsilon \sin 2\omega t\right) \\
&= \cos\omega t \cos\left(\varepsilon \sin 2\omega t\right) - \sin\omega t \sin\left(\varepsilon \sin 2\omega t\right) \\
&= \cos\omega t - \varepsilon \sin\omega t \sin 2\omega t + \dots\dots \\
&= \left(1 - \frac{\varepsilon}{2}\right)\cos\omega t + \frac{\varepsilon}{2}\cos 3\omega t + \dots\dots
\end{aligned}
$$

This is very similar to the solution of Duffing equation .

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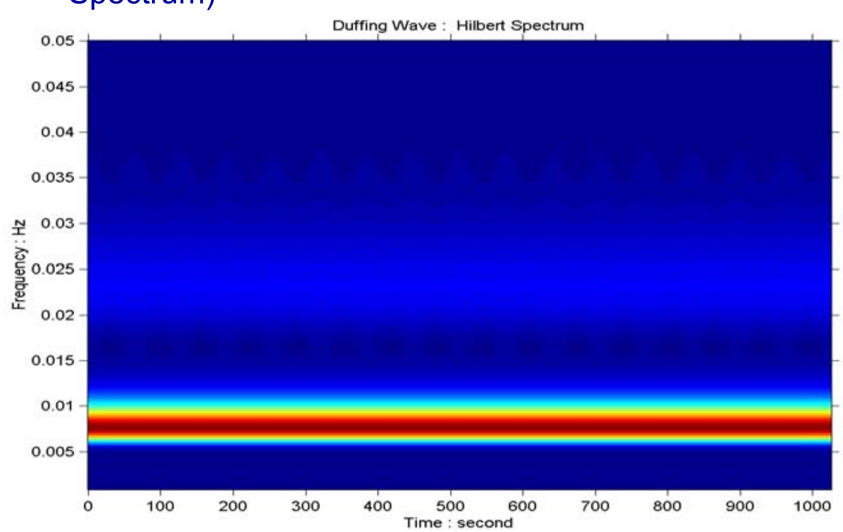
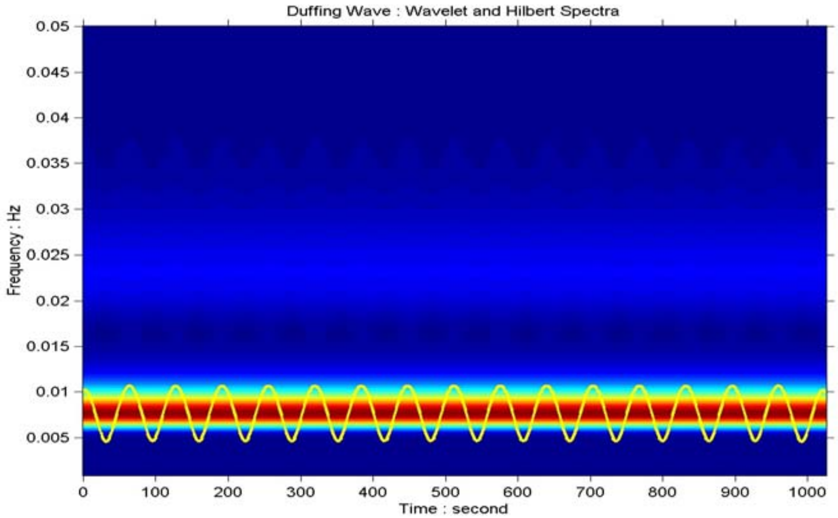
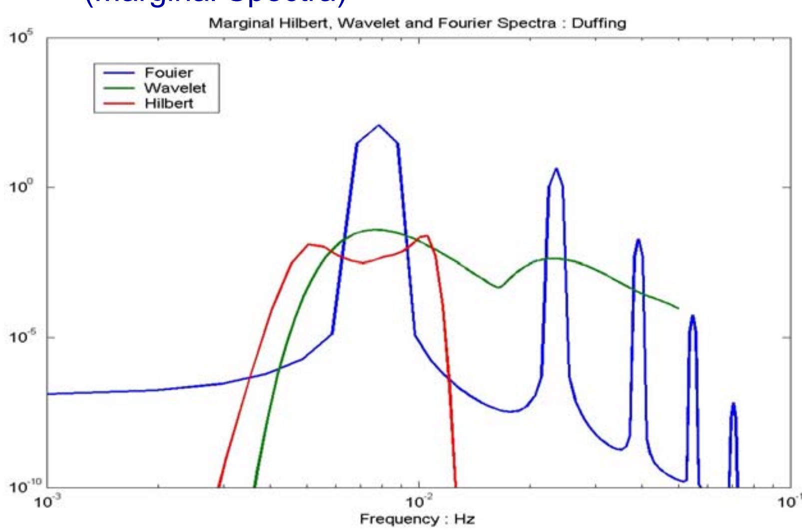
Nonlinearity: Duffing Type Wave (Wavelet Spectrum)



Nonlinearity: Duffing Type Wave (Hilbert Spectrum)



Nonlinearity: Duffing Type Wave (Marginal Spectra)



Nonlinearity: Duffing Equation

$$\frac{d^2 x}{dt^2} + x + \varepsilon x^3 = \gamma \cos \omega t \ .$$

Solved with ***ode23tb*** *for $t = 0$ to 200 with*

$\varepsilon = -1$

$\gamma = 0.1$

$\omega = 0.04\ Hz$

Initial condition :

$[x(o), x'(0)] = [1, 1]$

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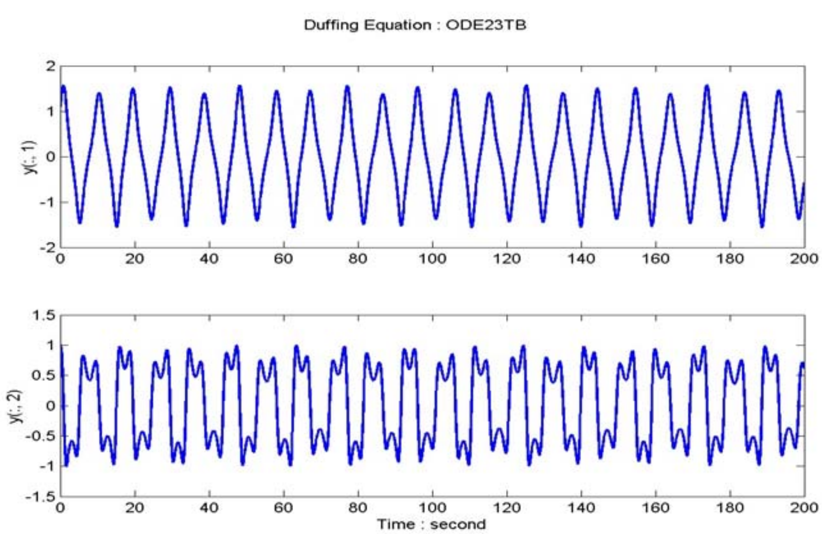
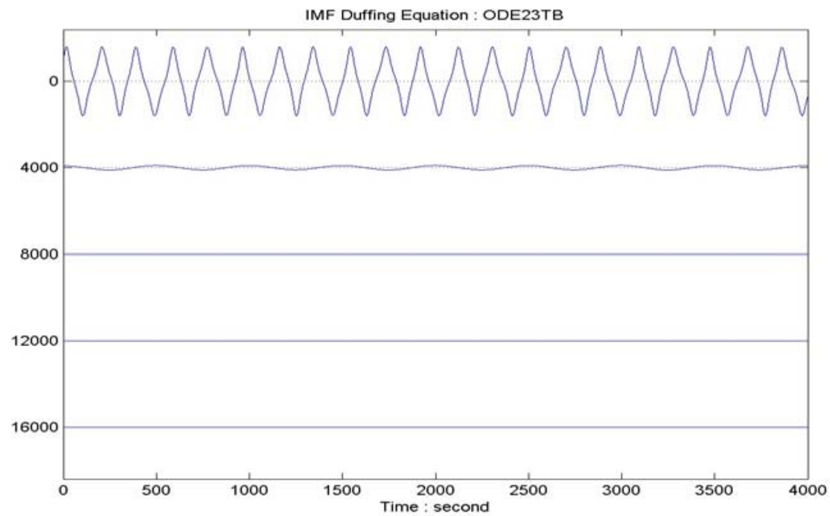
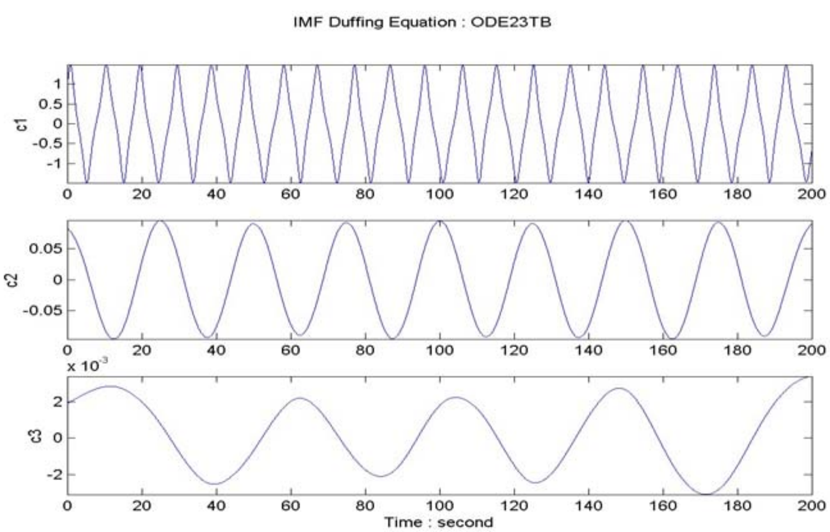
Nonlinearity: Duffing Equation (Data)



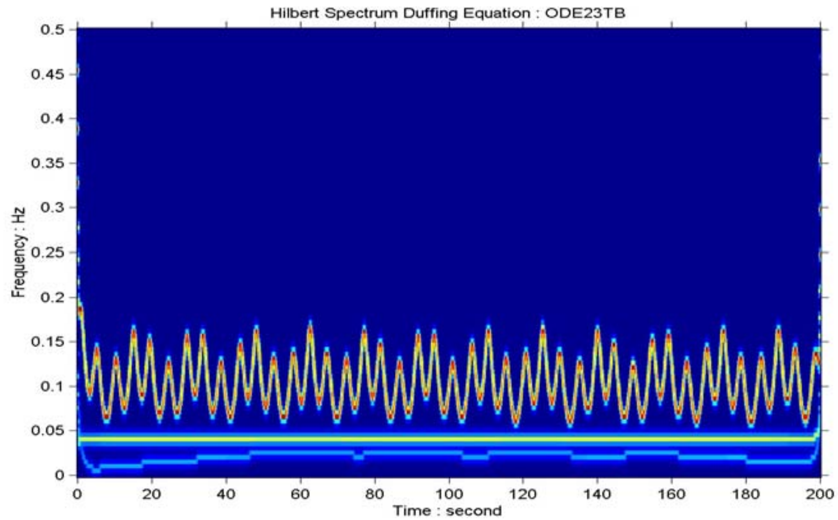
Nonlinearity: Duffing Equation (IMFs)



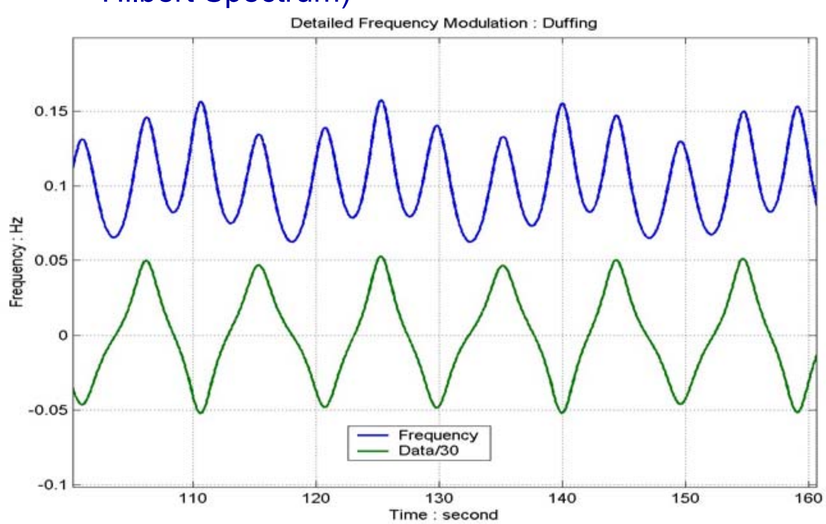
Nonlinearity: Duffing Equation (IMFs)



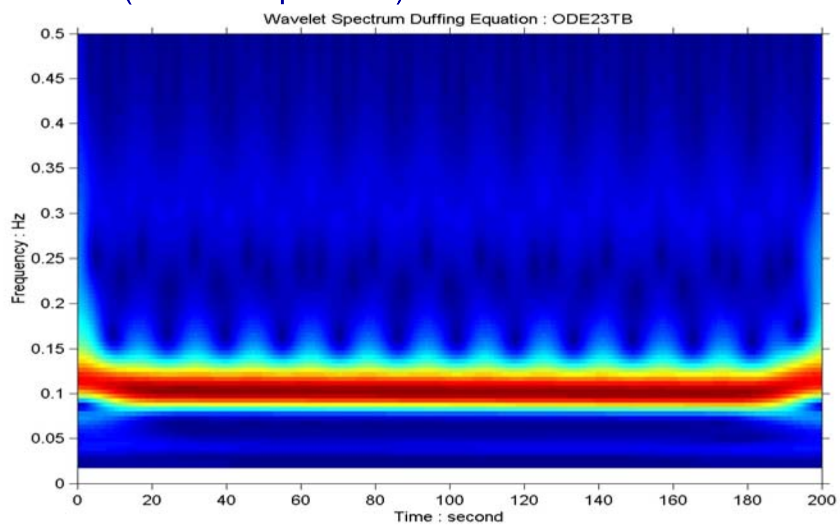
Nonlinearity: Duffing Equation (Hilbert Spectrum)



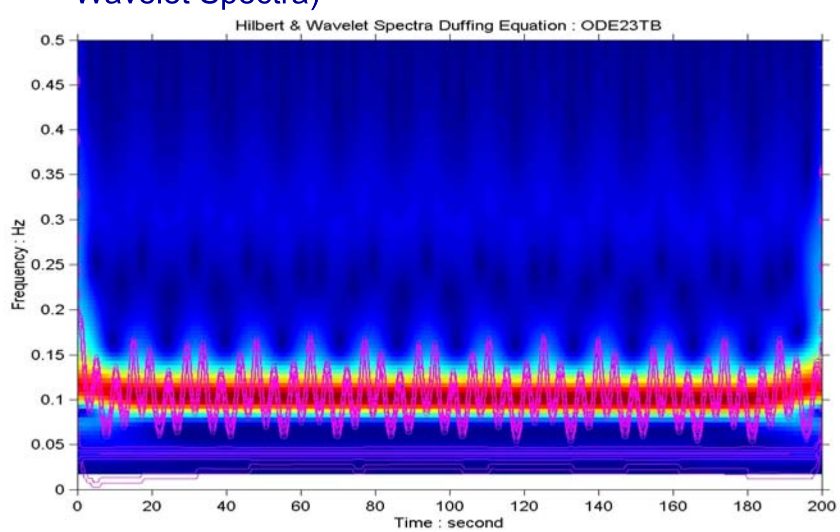
Nonlinearity: Duffing Equation (Detailed Hilbert Spectrum)



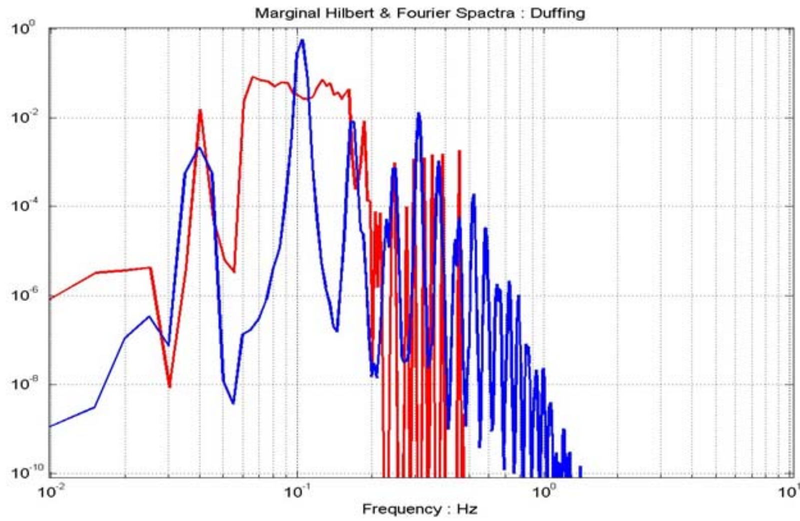
Nonlinearity: Duffing Equation
(Wavelet Spectrum)



Nonlinearity: Duffing Equation (Hilbert & Wavelet Spectra)



Nonlinearity: Duffing Equation (Marginal Hilbert Spectrum)



Nonlinearity: Rössler Equation

Rössler Equation solved with ode23 :

$$\dot{x} = -(y + z),$$
$$\dot{y} = x + \frac{1}{5}y,$$
$$\dot{z} = \frac{1}{5} + z(x - \mu).$$

Initital conditions :

$\mu = 3.5$

$[x, y, z] = [1, -1, 0]$

For

$t = 0 : 200$.

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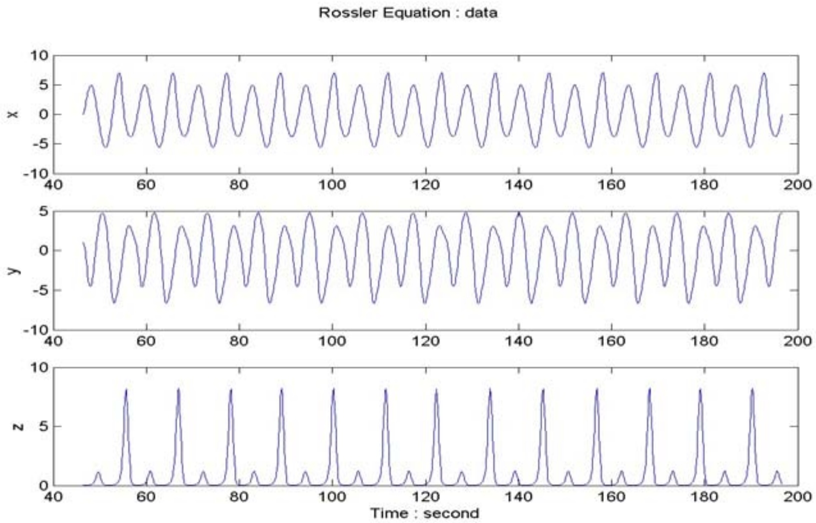
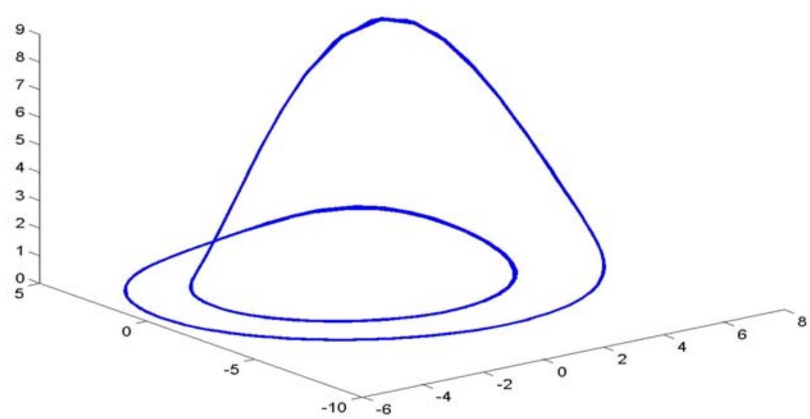
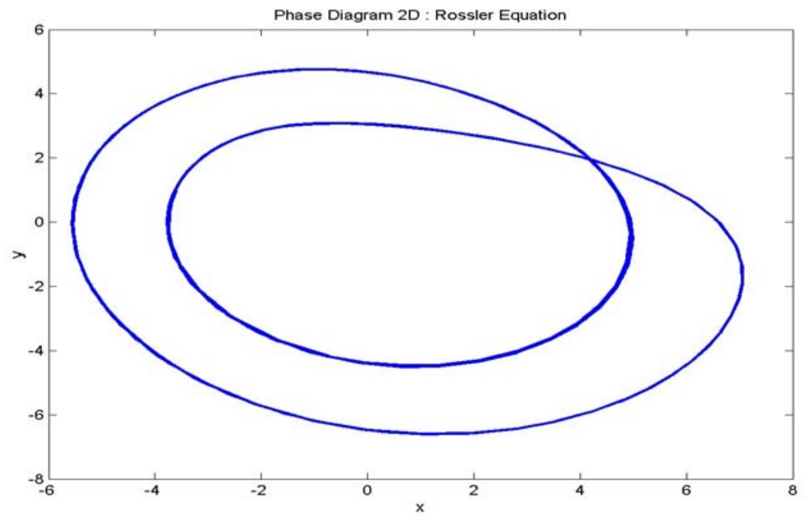
Nonlinearity: Rössler Equation (Data)



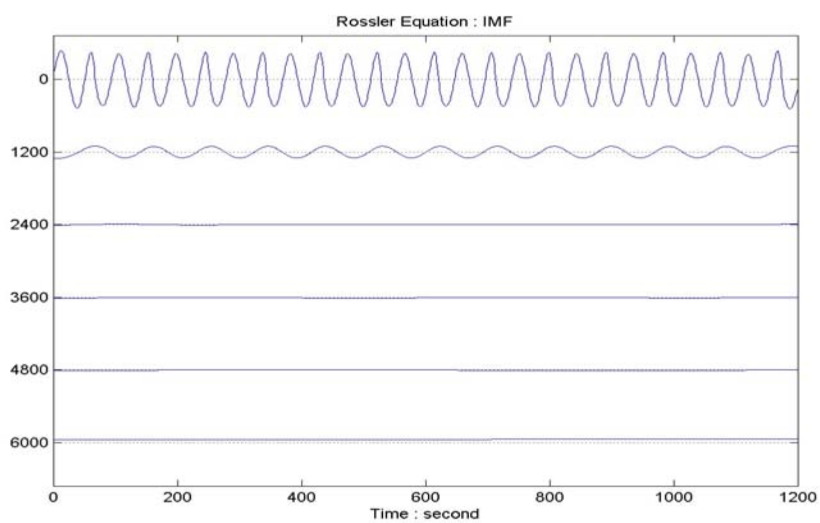
Nonlinearity: Rössler Equation (3D Phase)



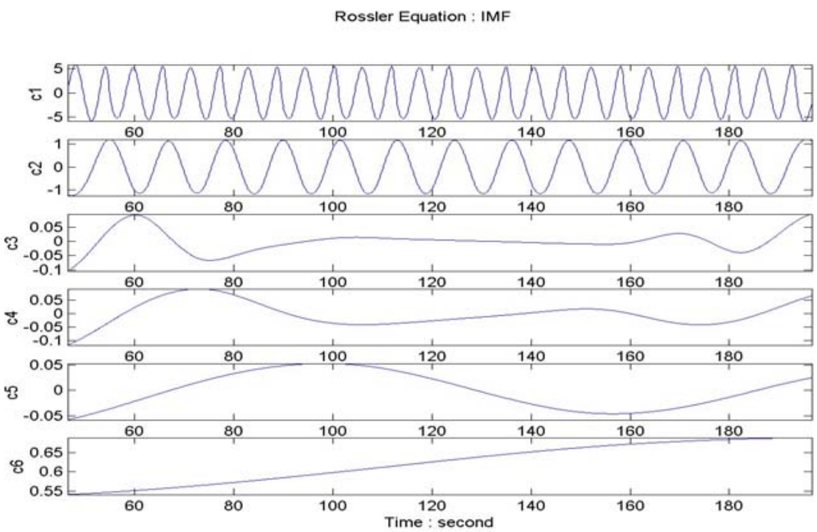
Nonlinearity: Rössler Equation (2D Phase)



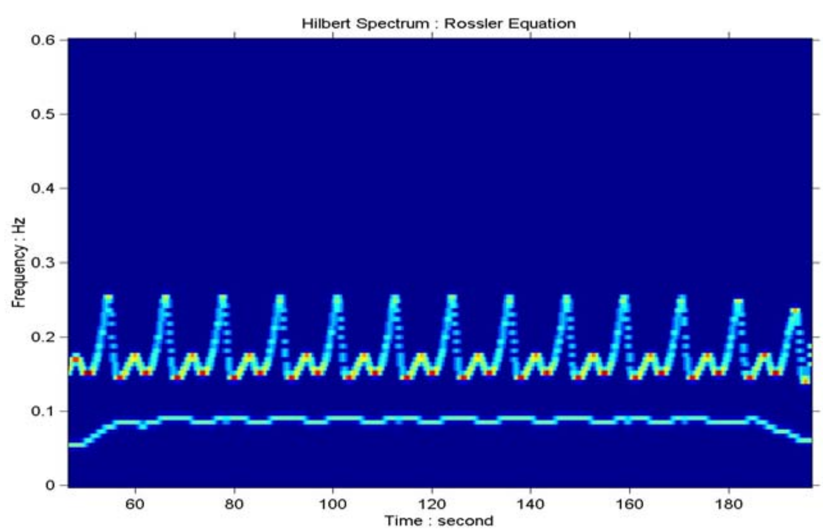
Nonlinearity: Rössler Equation (IMF Strips)



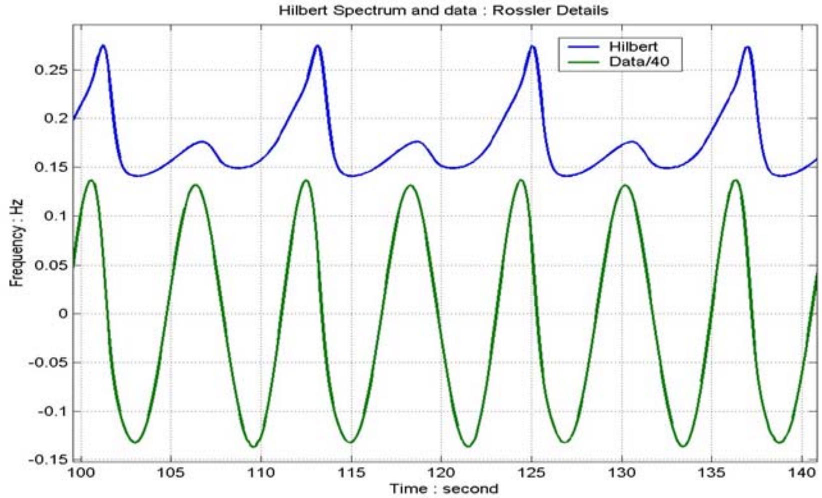
Nonlinearity: Rössler Equation (IMFs)



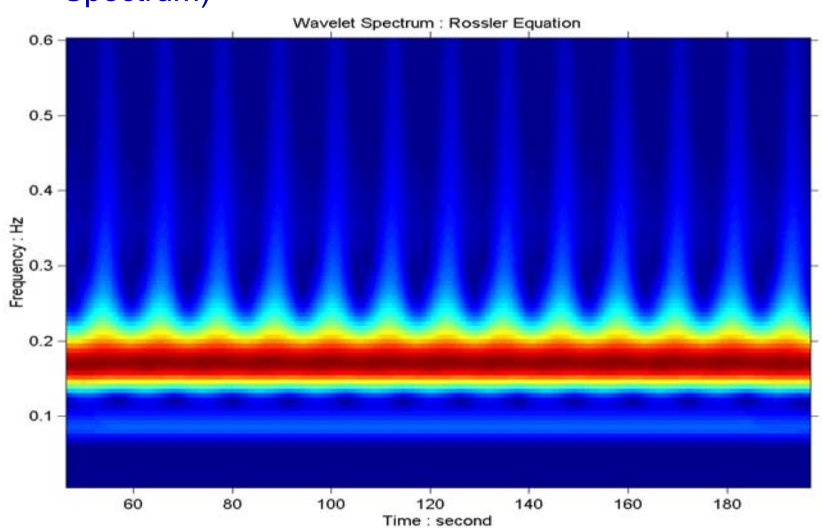
Nonlinearity: Rössler Equation (Hilbert Spectrum)



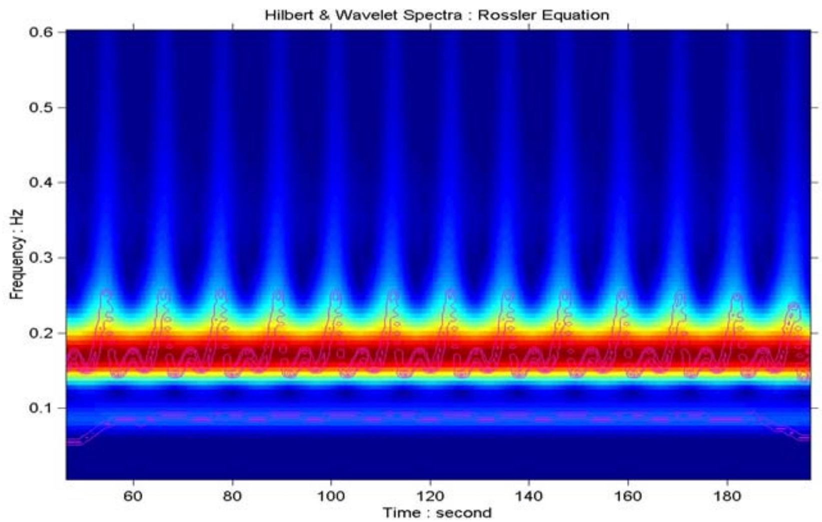
Nonlinearity: Rössler Equation (Hilbert Spectrum & Data Details)



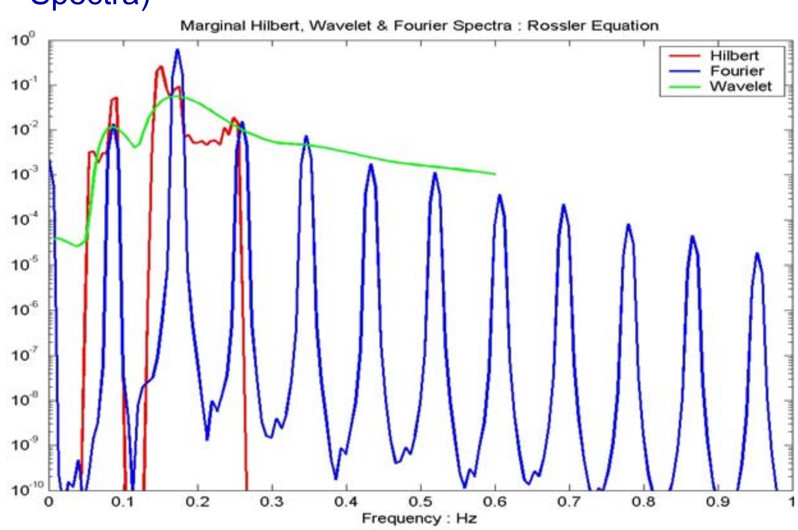
Nonlinearity: Rössler Equation (Wavelet Spectrum)



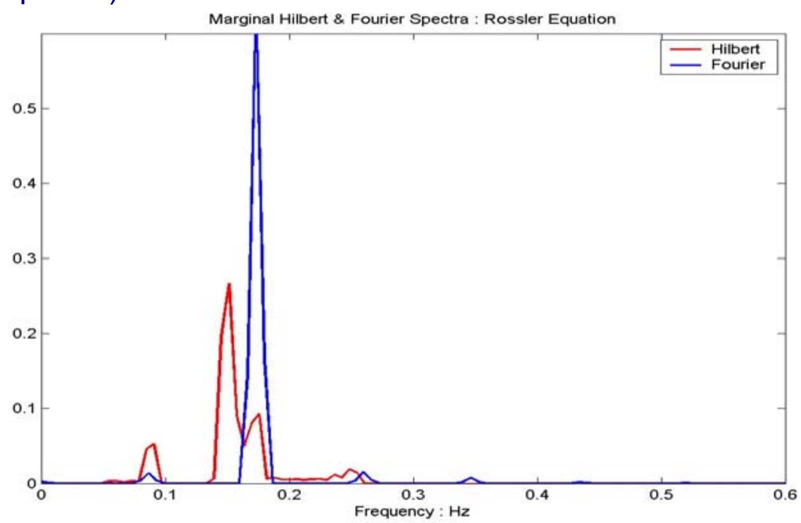
Nonlinearity: Rössler Equation (Hilbert & Wavelet Spectra)



Nonlinearity: Rössler Equation (Marginal Spectra)



Nonlinearity: Rössler Equation (Marginal Spectra)



Nonlinearity: What does this mean?

- Instantaneous Frequency offers a total different view for nonlinear data.
- An adaptive basis is indispensable for nonstationary and nonlinear data analysis.
- HHT establishes a new paradigm for data analysis.

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Nonlinearity: Comparisons

	Fourier	Wavelet	Hilbert
Basis	A priori	A priori	Adaptive
Frequency	Convolution: Global	Convolution: Regional	Differentiation: Local
Presentation	Energy-frequency	Energy-time-frequency	Energy-time-frequency
Nonlinear	No	No	Yes
Non-stationary	No	Yes	Yes
Feature extraction	No	Discrete : no Continuous: yes	Yes

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Nonlinearity: Different Paradigms Mathematics vs. Science/Engineering

Mathematicians

- Absolute proof
- Logic consistency
- Mathematical rigor

Scientists/Engineers

- Agreement with observations
- Physical meaning
- Working approximations

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Applications: Current Applications

- Non-destructive evaluation for health monitoring
  - (DOT, NSWC, and DRC/NASA, KSC Shuttle)
- Vibration, speech, and acoustic signal analyses
  - (FBI, MIT, and DARPA)
- Earthquake engineering
  - (DOT)
- Biomedical applications
  - (Harvard, UCSD, Johns Hopkins, and Southampton, UK)

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Applications: Current Applications

- Global primary productivity evolution time series from LandSat data
  - (NASA Goddard)
- Planet hunting
  - (NASA Goddard and Nicholas Copernicus University, Poland)
- Financial market data analysis
  - (NASA and HKUST)

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Examples: Airfoil Flutter Study

- The new NASA aeroelastic flight program is pushing the airfoil to a new frontier. HHT clearly identified the yield of the airfoil just before the final disintegration of the airfoil.

- Fourier totally missed the critical change.

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Examples: Location of the Test Wing

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Examples: Details of the Test Wing

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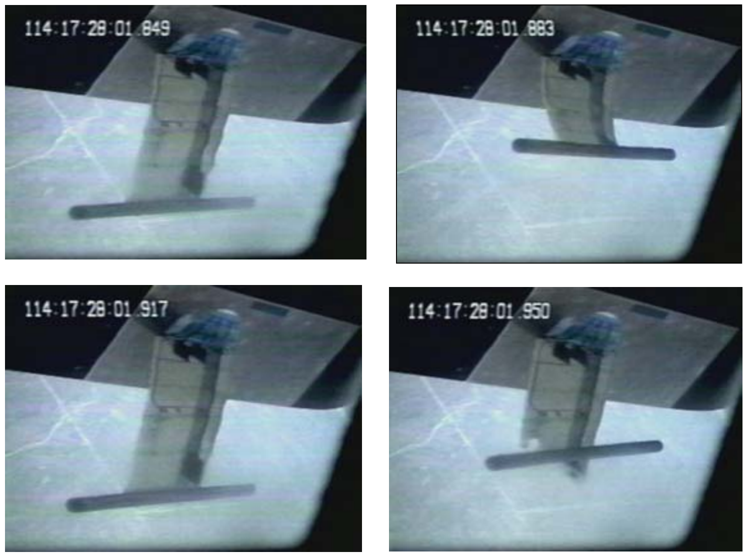
Examples: Airfoil Flutter



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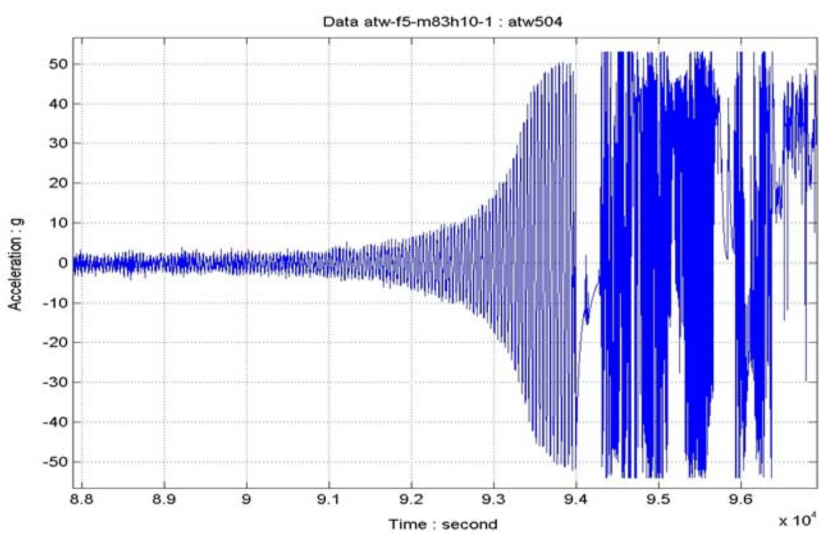
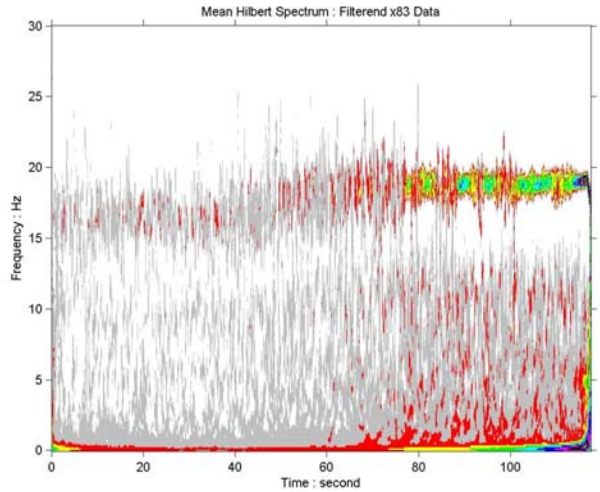
Examples: Full Data



Examples: Mean Hilbert Spectrum y(i)



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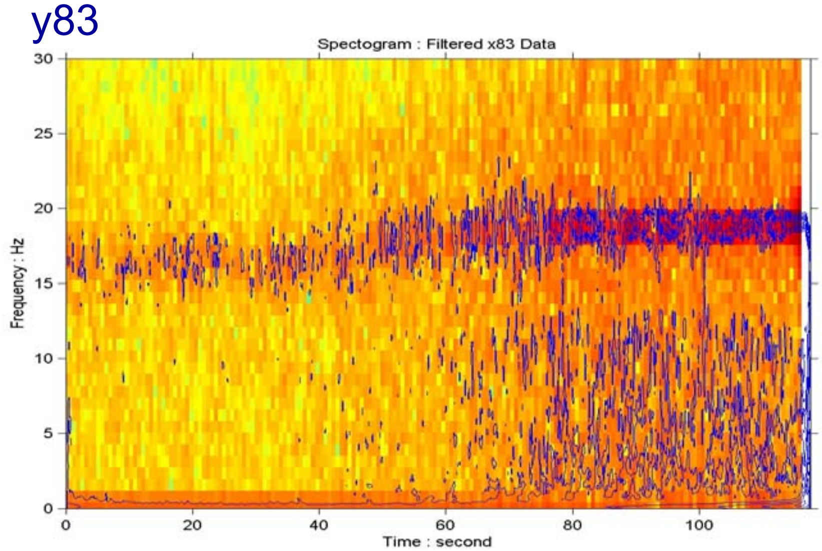
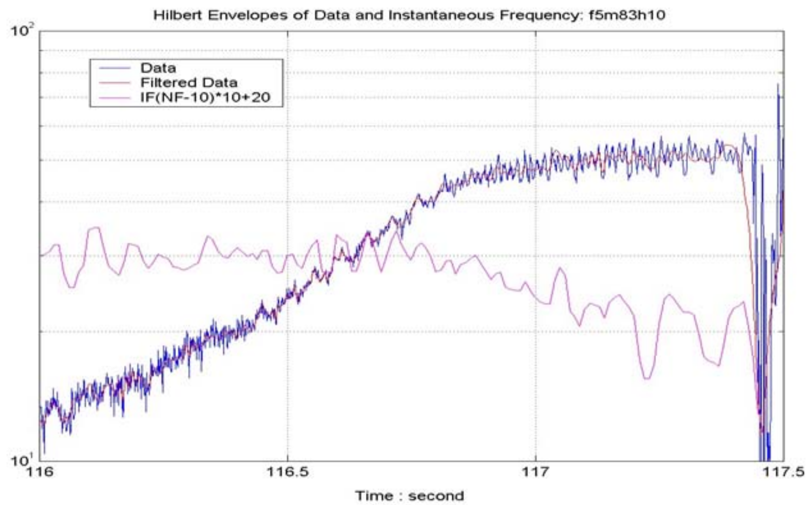
Examples: Mean Hilbert and Spectrogram y83



Examples: Instantaneous Frequency and Data Envelope



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Limitations: Limitations of Hilbert Transform

- Data need to be mono-component. Traditional applications using band-pass filter, which distorts the wave form. (EMD Resolves this problem)

- Bedrosian Theorem: Hilbert transform of $[a(t) \cos\omega(t)]$ might not be exactly $[a(t) \sin\omega(t)]$ for arbitrary a and ω . (Normalized HHT resolves this)

- Nuttall Theorem: Hilbert transform of $\cos\omega(t)$ might not be exactly $\sin\omega(t)$ for arbitrary $\omega(t)$. (Normalized HHT improves on the error bound).

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Unfinished Work: Outstanding Mathematical Problems

1.Adaptive data analysis methodology in general
2.Nonlinear system identification methods
3.Prediction problem for nonstationary processes
(end effects)
4.Optimization problem (the best IMF selection
and the issue of uniqueness, i.e. "Is there a unique
solution?")
5.Spline problem (best spline implementation of HHT,
convergence, and 2-D)
6.Approximation problem (Hilbert transform
and quadrature)

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Contact Information

If you are interested in learning more about NASA Goddard's HHT technology, please visit our Website:

http://techtransfer.gsfc.nasa.gov/HHT/HHT.htm

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